WALL PROPERTIES OF BRASS INSTRUMENTS

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ABSTRACT

The content of this thesis splits into two distinct areas, investigating the effect of material and its condition on the properties of brass instruments.

Firstly, acoustic losses in tubes are studied. A method for deducing losses from impedance and transfer function measurements is developed and applied to a number of tubes having different internal wall finishes. The cases studied are considered representative of the wall finishes found in brass instruments, both in factory condition, and after having been used for some time and not cleaned. Results show that a build-up of deposit on the inside of the tube causes the most significant increase in attenuation of sound propagating down the tube, whereas the results for smooth and roughened tubes are very close to theoretical predictions. An attempt is made to extend the theory to measurements on actual instruments but success is limited due to the flared portions of tubing.

Secondly, the vibration properties of trombone and trumpet bells are studied. A Finite Element package is used to model the bells and a post-processor program written to model the acoustic excitation and thus predict the forms of vibration. These calculated responses are compared with some results from experiments performed in conjunction with Southampton University.

The finite element model is also used to indicate the properties of the bell structure which are implicit in determining its mechanical response; supports (stays), material of construction, wall thickness, rim size and asymmetries in geometry.

It is difficult to give these properties musical significance though some results show that in certain circumstances, they do contribute to an instrument's musical characteristics.

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I am indebted to Dr. Ken Robinson and his support group at the Rutherford Laboratory for the speed with which the finite element work could be got underway and their help with coming to terms with the use of finite element packages on remote systems.

A special thank you is due to the Northam family who among them have kept me mobile, fed and have generally supported my welfare.

Thanks also to Mickey Fortuna for her excellent work in typing this thesis.

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SYMBOLS - Chapter 3

α	attenuation coefficient	neper m^{-1}
f	frequency	Hz
a	tube radius	m
B	a constant	
Z	series impedance per unit length	$\Omega\ m^{-1}$
Y	shunt admittance per unit length	$\Omega^{-1}m^{-1}$
V, Q, r_v, r_q	defined intermediates (section 3.2.1)	
Z_c	characteristic impedance	Ω (S.I. Ohms Kg $m^{-4}s^{-1}$)
γ	propagation coefficient	m^{-1}
ω	angular frequency	rad s^{-1}
ρ	density	Kg m^{-3}
c	velocity of sound in free air	ms^{-1}
η	viscosity	Kg $m^{-1}s^{-1}$
C_p	specific heat at constant pressure	J $Kg^{-1}K^{-1}$
κ	thermal conductivity of air	J m^{-1} K^{-1}
Γ	C_p/C_v	
j	$\sqrt{-1}$	
ρ_w	modified density of air	Kg m^{-3}
K	bulk modulus of air	Kg $m^{-1}s^{-2}$
K_w	modified bulk modulus of air	Kg $m^{-1}s^{-2}$
β	wave-number $\gamma = \alpha + j\beta$	m^{-1}
P_0	pressure at $x = 0$ (amplitude)	Pa

U_0	volume velocity at $x = 0$	m^3s^{-1}
Z_0	input impedance	Ω
ℓ	length of tube	m
P_ℓ	pressure at $x = \ell$ (amplitude)	Pa
Z_ℓ	termination impedance	Ω
P	pressure (general)	Pa
U	volume velocity (general)	m^3s^{-1}
A	a constant	
R	reflection coefficient	
T	transfer function	
G	a constant	
J_0, J_1	Bessel functions of the first kind	

SYMBOLS - Chapter 4

$W(x,y,z,t)$	general deflected shape of a structure	m
r	summation subscript	
q_r	generalised co-ordinate for r^{th} mode of vibration	m
$f_r(x,y,z)$	normalised shape for r^{th} mode of vibration	
M_r	generalised mass	Kg
C_r	generalised damping coefficient	Kg s^{-1}
K_r	generalised stiffness	Kg s^{-2}
L_r	generalised force	N
α_r	generalised receptance	
ω	angular frequency	rad s^{-1}
j	$\sqrt{-1}$	
μ	mass per unit area	Kg m^{-2}
x,y,z	cartesian co-ordinates	m
t	time	s
ρ	density	Kg m^{-3}
A_n	effective area at node n	m^2
T_n	thickness at node n	m
NN	number of nodes on structure	
η	damping factor	
P_0	pressure amplitude	Pa
$p(x,y,z)$	normalised pressure distribution	
f_n	normalised displacement normal to bell surface at node n	

CHAPTER 1. INTRODUCTION

In the spring of 1978 I was talking to a prominent trombone playing friend. After a thoughtful pause in the conversation he looked quizzical and said "I just don't understand how these sousaphones with fibre-glass bells work". I asked why and he replied "well, the brass goes round so far (indicating to somewhere behind his neck), then the rest is fibre-glass and that must be a poor conductor of sound".

Obviously he thinks that the sound is carried principally in the structure of the instrument rather than the air column, and although he may be an extreme case, he does reflect a surprisingly well respected view among even those who understand that the instrument is a contained volume of oscillating air. Opinion on the matter of the contribution to musical quality made by the instrument walls is diverse and certainly not lacking. There is a range from the misguided musician above to the staid scientist who refuses to consider that the walls could have any effect at all. Manufacturers tend to fall conveniently between the two extremes, probably trying to keep all parties happy; enemies do not buy instruments.

This thesis is concerned with rationalising the possible effect the walls of a brass instrument may have. The views of players, manufacturers, listeners and scientists are discussed, the scant scientific work to date is reviewed and possible ways in which the instrument walls could be formative of the sound produced are discussed.

The example of brass instrument used generally through the thesis is the trombone, a diagram of which is presented in figure 1.1, and a detailed diagram of a mouthpiece is presented in figure 1.2. A trombone is a good example for a number of reasons: most of the work of the research group at the University of Surrey has been concerned with the trombone, the pitch of the instrument is in the middle of the brass family, the tubing is of an "average" diameter (not as small as a trumpet or as large as a tuba), there are usually no valves causing peculiar discontinuities and the bell is of simpler geometry than most other brass instruments (see chapter 4: trombone vs. trumpet).

Experiments which investigate the effects of the internal condition of the instrument walls are then reported. Different wall finishes are idiosyncratic to methods of manufacture, some manufacturers intentionally polish the bore of an instrument, some players intentionally do not clean their instruments as they consider the sediment beneficial.

Finally there is a study of brass instrument bell vibrations and their response to acoustic excitation. This work is comprised of a mixture of experiment and calculations based on the finite element technique and generalised vibration methods.

It is hoped that the work in this thesis may provide a foundation from which further investigations may be refined and new investigations inspired. The subject involves much fundamental physics and many engineering problems, and though outwardly esoteric compared to much scientific work, a large industry may benefit from conclusive results.

Also, the work involved has pushed many physical and engineering techniques to new limits thus benefit may be felt in many other areas.

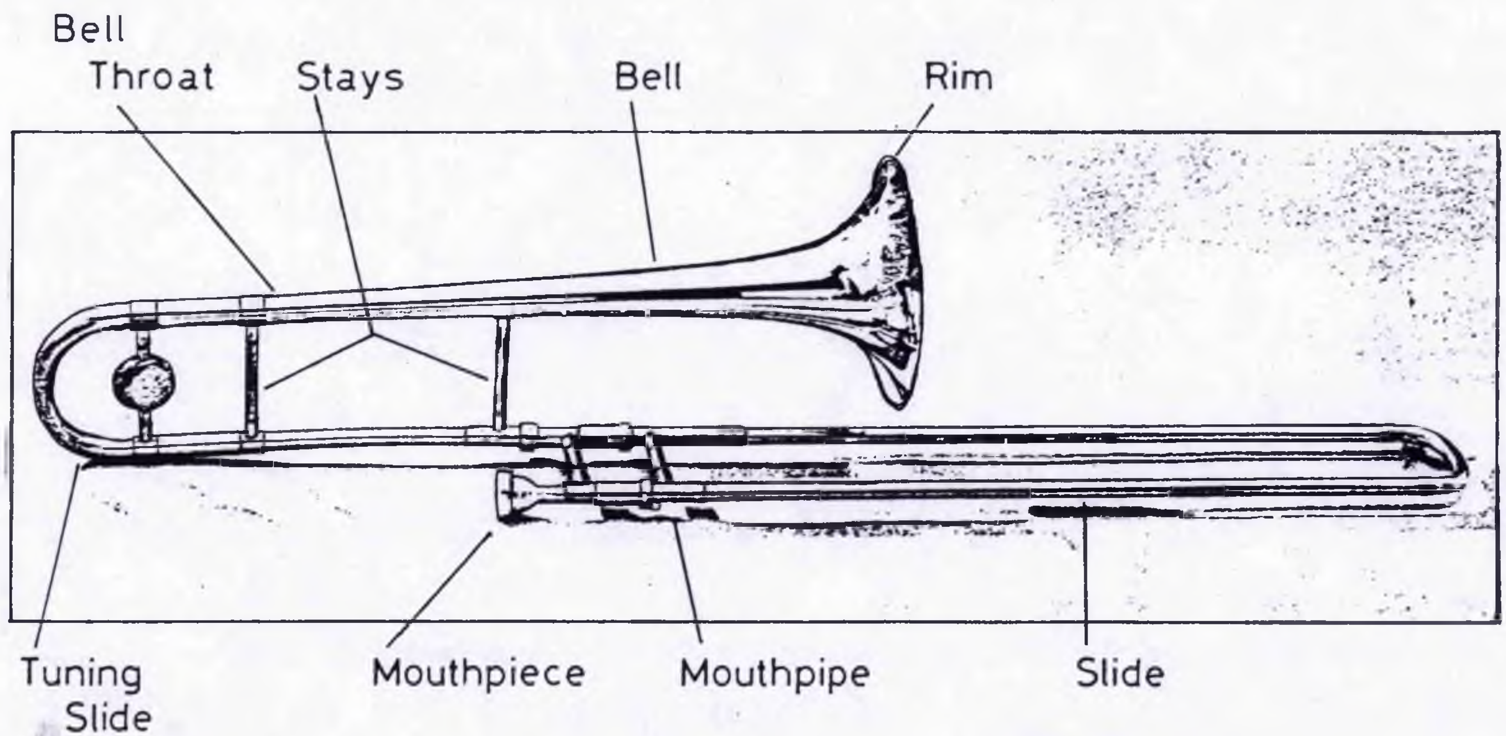


Figure 1.1 Construction of a Trombone.

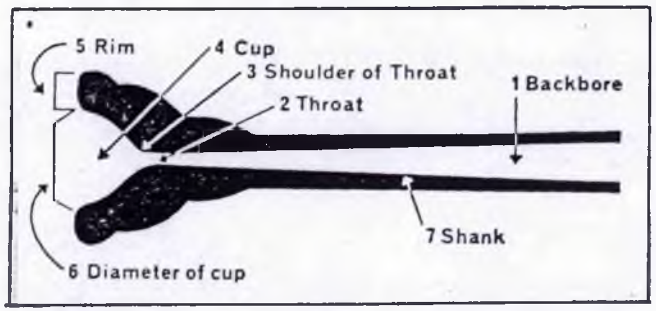


Figure 1.2 Construction of a Mouthpiece.

CHAPTER 2 : A REVIEW OF THE PROBLEM

2.1 Introduction

There are four quite distinct though not independent groups of people involved when assessing any quality of a musical instrument: the manufacturer, the listener, the player, and the scientist. With brass instruments there is much folklore related to the role of material in the endowment of tonal distinction; maybe some of it is justified, but certainly it has inflamed the emotions of the above factions to the extent that one will cast great doubt upon the credibility of another in the pursuit of their almost religious beliefs.

It is the purpose of this chapter to examine the positions and beliefs of these parties and then provide an unbiased approach.

2.2 Viewpoints

2.2.1 Manufacturer's view

The manufacturer's considerations split into three distinct subjects: development, production, and sales.

From the sales point of view, reputation comes first, (which can be difficult for a manufacturer to control), then it is the advertising campaign which constitutes the battleground for a share of the market. There is only a limited number of aspects of an instrument to which advantageous properties can be assigned, of which the material from which the instrument is made and its condition are used almost universally among makers to acclaim their products (see appendix (A)). Figures (2.1, 2.2) show examples of some of the advertising themes, not all directly citing material of constitution;

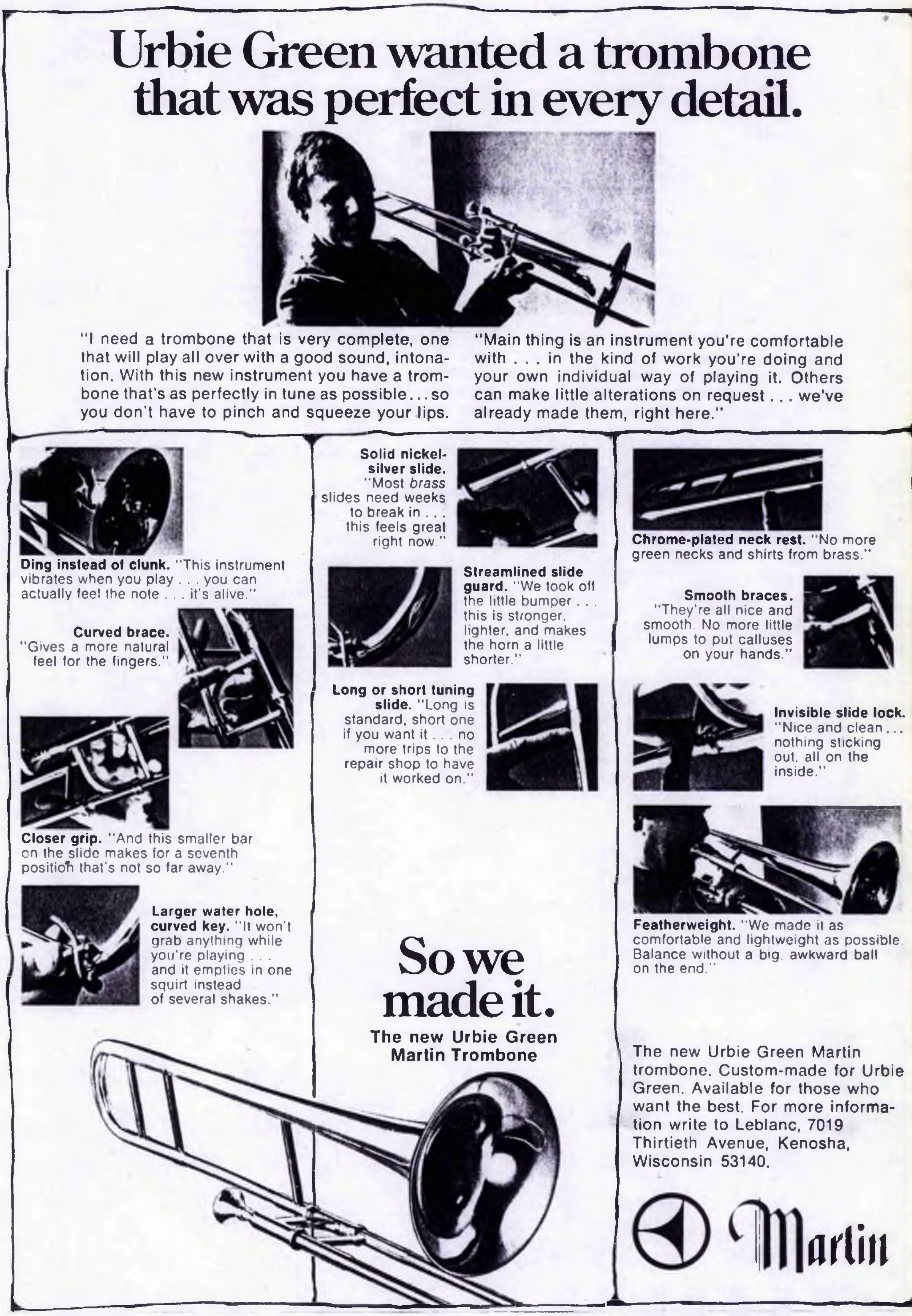


Fig. 2.1 BRASS INSTRUMENT ADVERTISEMENT

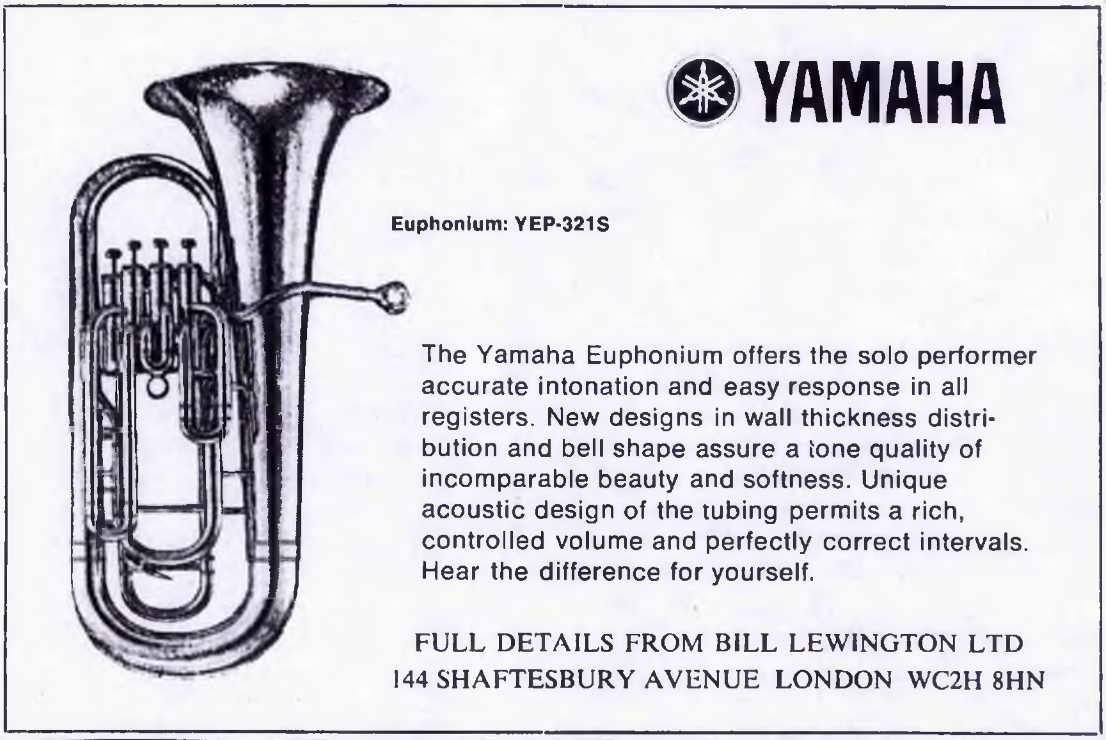


Fig. 2.2 BRASS INSTRUMENT ADVERTISEMENTS

the manufacturer's catalogue is the usual location of such details.

Considering production, materials must be of workable substance and dimensions, for example: the current "thin bell" school causes production problems due to the ease with which the thin brass tears under work and perforates when brazed. Examples of attempts to solve these problems are found in electroformed bells (produced by electro-deposition of material on a suitably shaped former), or bells thinned down by turning after fabrication from material of workable thickness. Instruments are further thinned in an undefined and unpredictable manner due to the surprisingly violent polishing process; harsh enough to cause severe damage if not performed with vigilant restraint. A more detailed account of the manufacturing process can be found in appendix (C). Having fabricated the instrument, it must then be durable. Even the most well cared-for instrument will wear appreciably, and knocks are often unavoidable.

The manufacturer's development processes are numerous and cover a wide range of philosophies and disciplines. The constraints on the developer are, as in all industry, those of development time, development and production cost, and manufacturability of the product. The market must also be taken into consideration i.e. current fashion and the prejudices of players.

Design procedures follow various techniques of modification linked with some sort of interactive subjective testing. Modifications vary from adjustments precipitated from the intuition of a craftsman, to bore perturbations calculated by computer (Smith and Daniell 1976). Subjective testing is performed by either professional musicians or a suitable permanent employee, and often both. The subjective testing methods vary from casual "tootling" by the subject and reliance on his casual judgment, to specific and structured subjective tests (Pratt and Bowsher 1978) yielding statistical data on the subjects appraisals and comparisons of the instruments under test. Generally though, the impression one gets is that the techniques employed in instrument development by manufacturers are not at all rigorous; not without some justification, though this apparent indifference may be due to manufacturers unwillingness to divulge details of their methods.

2.2.2 Player's view

Taking the catalogues to be representative of the manufacturer's opinions, then it is clear that most believe that material imparts certain musical qualities to their instruments. This belief is also found as constitutional in many players (Edwards 1978). Whether players have this belief seeded by manufacturer's advertising, or the manufacturer's advertising is prompted by the player's belief is an enigma lost in history but it is certain that the two factions now serve co-operatively to its propagation.

The player's ambition is towards better tools for his trade, not only searching for the best of those available, but also to produce new and superior instruments. Many top-line instruments have a player's name endowed to boost its credibility.

A player can be forgiven for intuitively bestowing musical qualities on the material of his instrument. The air column, although the major musical device, is a rather ethereal concept and not so easily related to such a personal possession. The player can often feel the instrument vibrating in his hands, which will extend his personification of his appliance from just his dynamic interaction at his lips.

Whether the effects are real or psychological, they exist in the player and therefore must be taken into consideration by the manufacturer in order to fulfil market demands.

2.2.3 Listener's view

Although the ultimate goal for the concern and work is to benefit the listener, it is the listener who has the least direct authority in the matter. Indirectly, it is the listener's criticisms that make a good player prominent compared to his contemporaries, but their judgment is principally based on skill and artistic interpretation rather than the substance of his instrument. It is quite generally accepted that a good player can still sound good even when playing a nominally bad instrument: it may be harder work, but the average listener would not be aware of this.

The player is a rather more active form of listener. It is the player who assesses and buys instruments, and uses what he hears to control the way in which he plays. Therefore it is much more lucrative to make considerations from the players stance rather than that of the passive listener, at least in the first instance.

2.2.4 Scientist's view

It is the habit of scientists to look for simple laws which govern the way in which physical systems behave. To do this, it is often the case that many assumptions are made in order to simplify the problem to a level at which only the factors considered "significant" are taken into account. The effect of the walls of brass instruments has been the subject of such an assumption which has probably caused more controversy than any other in musical acoustics.

Assumptions about acoustic waveguide walls are rooted in the work of Kirchhoff (1868) who, among other things, assumed that the gas in contact with the walls is at rest and that there are no irregularities in the walls of the waveguide of sufficient size to produce appreciable irregular motion in the gas. Thus, scientific experiments based on his theory (which accounts for all, almost without exception) use tubes of smooth bore and thick walls (e.g. Henry 1931, Shields et al. 1965, Fay 1940, Weston 1953). It is not generally stated in scientific writings that material has definitely no effect on qualities such as timbre and responsiveness, but that the material has no effect providing that the walls are reasonably thick, rigid, massive, etc. (Richardson 1953). Much of the work has dealt with only perfunctory testpieces and little effort has been directed towards finding the points at which wall material, condition, and thickness become significant.

Also, scientists are often unsympathetic towards player's and manufacturer's demands, probably due to communication barriers and narrow-mindedness. Players are annoyed by the scientists's disbelief in their subjective appraisals and conclusions; due to the lack of understanding of the need for objective evidence. Scientists are annoyed by the players' misinterpretations of their conclusions; due to the lack of appreciation of the careful, qualified wording of science.

2.3 Critique

Literature directly involving the materials of brass instruments is sparse. Many more general works acknowledge a possibility of wall effects but they usually either leave the question open or assume the effects negligible with reference to one or more of the appropriate papers mentioned below. There are also a number of papers with a lesser degree of relevance, namely those dealing with organ pipes and wood-wind instruments.

Miller (1909) relates the tale of how impressed he was with the quality of a particular flute made from gold. His subjective appraisals convinced him that this was due to some acoustic advantage that gold has over silver or wood. His paper includes a review of pre-1900 opinion on the subject, mentioning such names as Boehm, Mahillon, and Schafhäutl (an "eminent professor at the University of Munich"). Miller's experiments were with rectangular organ pipes fabricated from wood and zinc. His results were observations of the acoustic behaviour of the zinc pipes when squeezed with the hand and when enclosed by a water jacket of variable depth. Many distinct changes of tone and pitch were noted and it was concluded that these changes prove a dependence on wall material. However, Backus (1964A) points out that the effects are due to the pipes being rectangular, and that the corresponding effects in a cylindrical pipe would be negligible. Backus cites the results of Boner and Newman (1940) to verify this.

The work of Boner and Newman (1940) has caused much dispute. They compared the steady-state spectra produced by a number of cylindrical tubes, each of different material and/or wall thickness, each of which slotted onto a common exciter (an organ pipe foot). Small differences were found but they concluded that the differences were negligible. Backus uses this conclusion to his advantage but is contrasted by Sumner (1962) who uses the same results to conclude that the organ builders positive views on consequences of material choice are confirmed.

A commonly referenced paper is that of Knauss and Yeager (1941). By measuring sound output from a cornet excited alternately acoustically and mechanically they conclude that the sound radiated from the walls is negligible compared to that radiated by the air column. In addition to this they performed a playing test comparing the unfettered instrument, the same instrument with putty on the inside(?) of the bell, and the instrument with putty on the outside of the bell. Discounting the case with putty on the inside of the bell (i.e. a change of bore shape) then "very little difference in tone quality could be noted". The work described is very inconclusive: only one instrument was tested, the assumed masking levels are for pure tones of the same frequency so no account is made of possible harmonic distortion due to radiating walls, and the position of the microphone relative to the cornet for the sound radiation measurements is not discussed therefore not accounting for possible directional and proximity effects.

Mercer (1951) is rather non-committal about material but writes that the wall thickness is critical - if the walls are too thin then the pipe cannot produce a powerful note, if the walls are too thick then the tone produced lacks "life". No evidence is offered to substantiate this, though there is a reference to the Boner and Newman (1940) paper discussed above.

Benade (1959) considers the magnitude of wall losses in woodwind instruments citing the results of Fay (1940) for attenuation in brass tubes and presenting some rough empirical attenuations for plastic tubes with and without fingerholes. The effect of wall material on tone quality is discussed inconclusively and no new evidence is presented.

A similar but more thorough investigation to that of Knauss and Yeager was performed by Backus (1964B) working with a clarinet. He found that the sound radiated from the instrument walls was insignificant compared to that radiated from the air column and that to detect any radiated sound from the walls at all, the microphone had to be placed very close to the instrument. Also the sound spectrum emitted from a brass tube fitted with a reed was compared to that of a plastic tube fitted with the same reed. It was found that both provided a "similar" spectrum when blown.

This paper was followed by a theoretical and experimental examination of organ pipes (Backus and Hundley 1966). Compliance of the pipe walls due to internal pressure oscillations, the resultant sound radiation, and the resultant modification to the velocity of sound in the tube is given a simple theoretical treatment, and estimates of the magnitudes of these effects are made for pipes of circular, elliptical, and square cross section. It was found that the radiation will be insignificant for a typical case, though this is not conclusive as resonance of tube walls is not taken into account in the theory. It is also found that the oscillation of cross sectional area due to complying walls is proportional to a/t (a = internal radius, t = wall thickness) for the cylindrical pipe but proportional to $(a/t)^3$ for the elliptical and square pipe. Damping is not included in the theory.

The experimental work reported is similar to that of Miller (1909), though a cylindrical pipe is compared to a square pipe. When enclosed in water the square pipe was found to change pitch, but "little difference" was noticed with the cylindrical pipe. Measured sound radiation from the pipe walls was found to be insignificant though the amplitude of wall vibration was considerably higher than that predicted by the theory presented earlier in the paper. This is attributed to the mechanical response of the lip due to the air stream impinging upon it rather than to a resonance in the air column.

Benade and Gans (1968) discuss various loss mechanisms with particular attention paid to viscous and thermal losses. The paper of Benade (1959) is cited for experimental data, the only relevant addition being some results from "rough measurements" on a clarinet having its bore polished to successively finer degrees between measurements. Although the text is directed towards woodwinds, it is stated that the various effects mentioned are essentially applicable to brass instruments.

Some experiments were carried out by Coltman (1971) who subjectively investigated the effect of material on the tone quality of flutes. Keyless (no fingerholes) flutes of silver, copper, and wood were compared. Each had a similar plastic head and all were mounted in a plastic bowl such that only the plastic head was felt or seen by the player. Subjective trials were then performed on both players and listeners, concluding that nothing could be distinguished between the three flutes under test. This work is hindered by the artificial nature of the test instruments.

Benade (1976) talks about possible material influences mentioning manufacturing problems and effects due to porosity of walls, internal damping in the walls, and turbulence of sharp edges, though no evidence is provided to indicate the significance of any of these factors.

Recent work has been done and reported by Smith (1978). Mechanical resonances of trombone bells have been studied holographically and the dependence of these on wall thickness determined. The effect of these vibrations is intuitively linked with improved response of the upper register of the instrument. Subjective experiments have been performed comparing three instruments: one with a thick (0.5 mm) brass bell, one with a thin (0.3 mm) brass bell, and one with a fibreglass bell, all of the same internal dimensions. No difference could be discerned by listeners and players between the thick brass bell and the fibreglass bell, but there was a quite noticeable difference between the thin brass bell and the fibreglass bell.

The above selection of papers comprises the fundamental scientific literature directly related to the problem under consideration. The foundations provided by their contents are inconclusive, inconsistent, and incomplete, leaving many basic problems to be formalised and solved.

2.4 Analysis

To understand how material may affect the tone quality of a brass instrument, it is initially necessary to examine the ways in which a player interacts with his instrument. Fig. (2.3) is a representation of the physical and psycho-acoustic information flow between player, instrument, and environment. Below is a discussion of the important and relevant facets of the picture.

2.4.1 Visual

The colour and general appearance of the instrument will affect the players attitude from a cosmetic standpoint, and at a deeper level due to synaesthesia (Birren 1978). Scientifically this is a rather

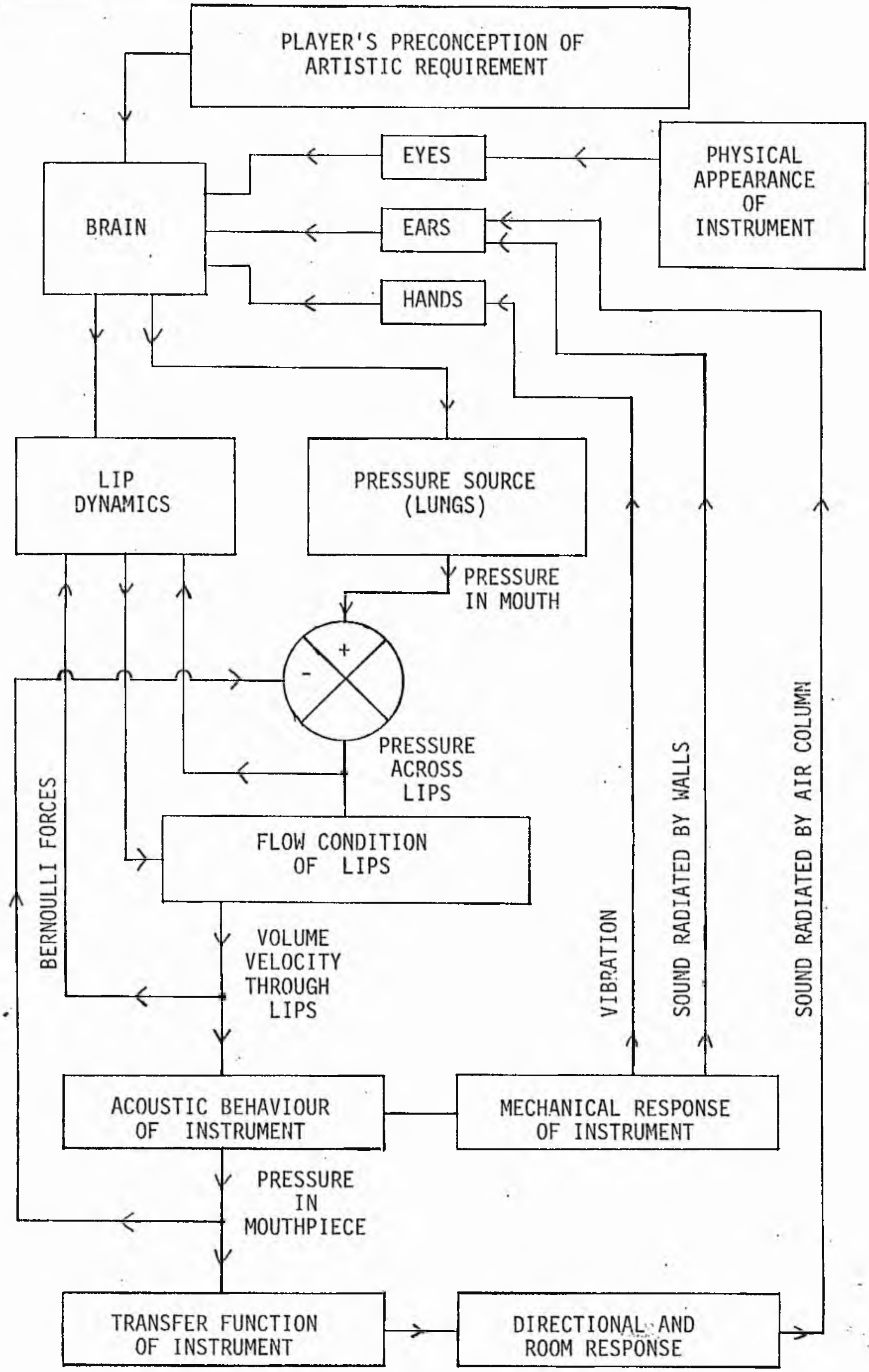


FIGURE 2.3 PLAYER'S INFORMATION FLOW DIAGRAM

amorphous subject and will certainly be swamped by the personal taste of the player.

2.4.2 Palpable

The instrument will present a texture to both the player's hands and his lips. This will be dependent on the type of finish given to the instrument and its effect is governed by the personal taste of the player.

All instruments will mechanically vibrate to some degree and it is often the case that these vibrations will be of sufficient level to be detected by touch; again both to the hands and also to where the lips are in physical contact with the mouthpiece. Each instrument will have its own characteristics of vibration and it is unknown how these characteristics will affect the player's appraisal of an instrument and also how they will interfere with the control the player exercises over his performance.

2.4.3 Direct aural

The vibrating walls of an instrument will radiate sound. The amount of radiation may or may not be significant. Though published experiments show that the radiation level is insignificant (Knauss and Yeager 1941, Backus 1964B, Backus and Hundley 1966), they are not comprehensive and do not specifically treat the case of, for example, the trombone player where the directional properties of sound radiation from the air column must be considered, and also that the bell passes within inches of the player's ear. It is quite conceivable that this extra sound output will not be detected by the listener, but will colour the player's perception of his own sound who will adjust accordingly.

The material and wall thickness will have bearing on the characteristics of this secondary radiation.

2.4.4 Physical acoustic

Under this heading are grouped physical effects such as loss due to viscous drag at the walls, thermal loss through the walls, turbulence, wall vibrations, leaks etc.; all of which will influence the tone quality, and also the nature of the reflected waves which interact with the players lips.

It is to the considerations of sections 2.4.3 and 2.4.4 that the major part of this thesis is devoted.

CHAPTER 3 : ACOUSTIC LOSSES IN BRASS INSTRUMENTS

3.1 Introduction

One of the factors pertaining to brass instrument walls is the nature of the surface presented to the air column. This internal surface accounts for viscous and thermal loss effects which are reviewed and discussed in this chapter, and also less tractable effects such as turbulence. A wide variety of internal conditions is met in brass instruments and range from the highly polished "micro finish" presented by Conn, through instruments with no particular internal finish, to instruments which have been played for years and never cleaned.

The effect of such finishes is investigated in this thesis using two fundamental acoustic measures: Input Impedance (Z_0) which is defined as input pressure divided by input volume velocity, and Pressure Transfer Function (T) which is defined as output pressure divided by input pressure (where the "output" is the termination of the acoustic waveguide and "input" is the acoustically driven end of the waveguide). The terms "impedance" and "transfer function" imply an analogy with two port electrical networks (Elliott 1979), and "waveguide" implies the further analogy of modelling the network with Transmission Line parameters. The investigation is based on these analogies and the fundamental equations of the general transmission line in acoustic terms can be found in appendix (B).

3.2 Acoustic Loss Mechanisms

3.2.1 Viscous and thermal losses

Theories of sound propagation in tubes, taking into account viscous and thermal effects have been expounded and reviewed for a number of years. Stokes (1845) is recognised as the initiator of the theory to cover viscous losses, followed by Kirchhoff (1868) who included thermal energy loss effects. The theory was reviewed by Rayleigh (1894), and much later a thorough review was presented by Henry (1931).

Henry shows that for practical rigid and smooth tubes, the basic assumptions of zero axial particle velocity at the walls, and the gas near the wall being at the same temperature as the wall (this temperature being constant), are valid and allow good theoretical predictions. In fact, he shows that the way in which these assumptions break down causes a smaller attenuation than predicted whereas experimental results tend to show larger attenuations.

Weston (1953, 1980) reviews the equations with emphasis on the quality of approximate solutions pertaining to particular ranges of parameters, and Shields et al. (1965) give a numerical solution of the fundamental equations to compare with experimental and analytical results. There are many more examples in the literature of considerations of these effects e.g. Crandall (1927), Zwikker and Kosten (1949), Daniels (1950), Tijdeman (1975), Shteinberg (1976), all of which serve to solidify the principles laid down by Kirchhoff (1868).

Experimentally, the most notable results are those of Fay (1940). The first approximation of the theory for the case under consideration (viz. the tube radius much smaller than the wavelength of sound propagating within that tube) is an attenuation which is inversely proportional to the tube radius and proportional to the square root of frequency:

$$\alpha = G \sqrt{f}/a \tag{3.1}$$

Fay measures the constant G to have the value of 2.92 (S.I. units), and also finds a weak dependence proportional to frequency.

Considerations of viscous and thermal losses in wind instruments are made by Benade (1959), who quotes equation (3.1) in a slightly different form. Values for G derived from "rough measurements" on plastic tubes with and without fingerholes are discussed and found to be significantly higher than the theoretical predictions and the results of Fay.

Benade and Gans (1968) assert that the viscous and thermal loss effects are the dominant sink for energy in most wind instruments, though they reference Benade (1959) for data.

Nederveen (1969) includes viscous and thermal loss in his analysis of woodwinds by modifying the density and bulk modulus of the gaseous medium, the modification is as follows:

$$\rho_w = \rho[1 + (1 - j)/r_v] \tag{3.2}$$

$$K_w = K[1 - (1 - j)(\Gamma - 1)/r_q] \tag{3.3}$$

where

$$r_v = \left(\frac{\omega\rho}{\eta}\right)^{\frac{1}{2}}.a \qquad r_q = \left(\frac{\omega\rho\ C_p}{K}\right)^{\frac{1}{2}}.a$$

$\Gamma = C_p/C_v$

These modified formulae are referenced to Kirchhoff (1868), Crandall (1927), and Zwikker and Kosten (1949). This technique of accounting for losses is also used by Jansson and Benade (1974).

To calculate viscous and thermal losses, Backus (1975, 1977) uses approximate solutions to equations quoted by Benade (1968), which are based on the form presented by Crandall (1927) and Daniels (1950). These "exact" (sic.) equations are expressed in terms of the transmission line distributed parameters and are as follows:

$$Z = j\left(\frac{\omega\rho}{\pi a^2}\right)(1 - V)^{-1} \tag{3.4}$$

$$Y = j\left(\frac{\omega\pi a^2}{\rho c}\right)(1 + (\Gamma - 1).Q) \tag{3.5}$$

where

$$V = \frac{2}{r_v(-j)^{\frac{1}{2}}} \cdot \frac{J_1[r_v(-j)^{\frac{1}{2}}]}{J_0[r_v(-j)^{\frac{1}{2}}]}$$

$$Q = \frac{2}{r_q(-j)^{\frac{1}{2}}} \cdot \frac{J_1[r_q(-j)^{\frac{1}{2}}]}{J_0[r_q(-j)^{\frac{1}{2}}]}$$

Equations (3.4) and (3.5) are more conveniently expressed in terms of the transmission line parameters Z_c and γ:

$$Z_c = \left[\frac{Z}{Y}\right]^{\frac{1}{2}} = \frac{\rho c}{\pi a^2}\,[(1 - V).(1 + (\Gamma - 1).Q)]^{-\frac{1}{2}} \tag{3.6}$$

$$\gamma = [Z.Y]^{\frac{1}{2}} = j\,\frac{\omega}{c}\left[\frac{1+(\Gamma-1).Q}{(1-V)}\right]^{\frac{1}{2}} \tag{3.7}$$

Note that $\frac{j\omega}{c}$ is the wave-number in free air and that ρc is the characteristic impedance of free air.

Equations (3.6) and (3.7) have been evaluated over the frequency range 10 Hz to 1024 Hz using a digital computer and the results are plotted in figures (3.1) and (3.2) for a tube of radius 0.0055 m. Their approximate counterparts (equations (3.8), (3.9) and (3.10)) are also plotted for comparison:

$$\gamma = \alpha + j\beta$$

$$\alpha = 3{\cdot}0 \times 10^{-5}\sqrt{f}/a \tag{3.8}$$

$$\beta = 2\pi f/c \tag{3.9}$$

$$Z_c = \frac{\rho c}{\pi a^2} + j0 \tag{3.10}$$

Very little difference can be discerned between the exact and approximate solutions, though differences between impedance and transfer functions calculated from either are significant (see section 3.2.4 and figures (3.3) and (3.4)).

3.2.2 Surface roughness and turbulence

Among Kirchhoff's assumptions on which the equations in section 3.2.1 depend are that the gas in contact with the walls is at rest, and that there are no irregularities in the walls of sufficient size

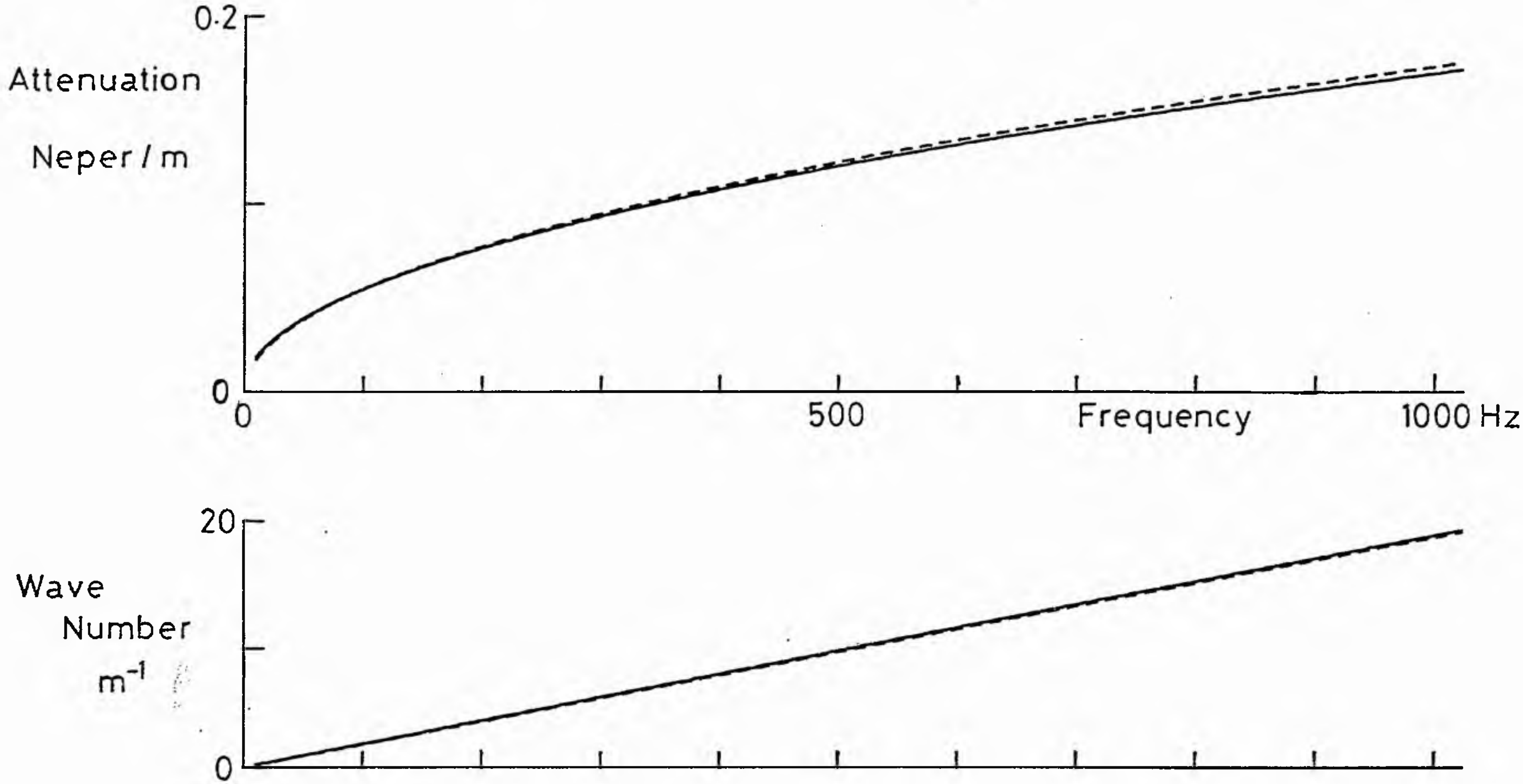


Figure 3.1 Theoretical propagation coefficient for a cylindrical tube radius 0.0055 m "Exact" (solid) and approximate (dashed).

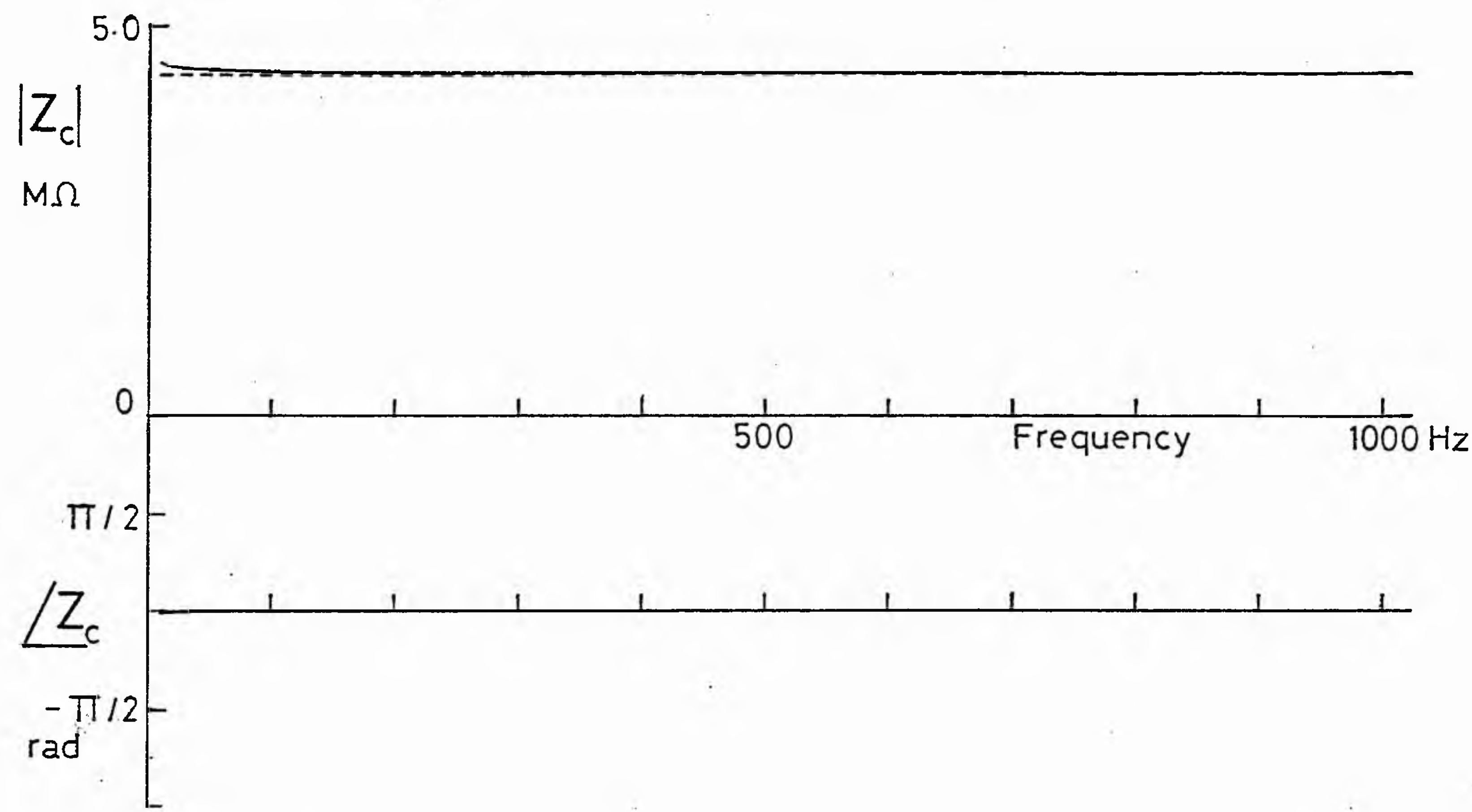


Figure 3.2 Theoretical characteristic impedance for a cylindrical tube radius 0.0055 m "Exact" (solid) and approximate (dashed).

to produce irregular motion of the gas (Henry 1931). In the laboratory, tubes can be chosen to approximate these assumptions. The tubing used in brass instruments does not necessarily meet these requirements.

Benade and Gans (1968) considering woodwind instruments say that the viscous and thermal losses are dominant, quoting a figure of 75% of the total energy loss (the other 25% being attributed to vocal tract, reed, turbulence and radiation losses). They give a result derived from "rough measurements" of a total fall of "as much as 10%" as the bore of a clarinet is polished by successive passages of a swab.

Henry (1931) theoretically considers slip at the tube wall and makes a correction in the viscosity term of Kirchhoff's formula to account for it. Weston (1953) considers irregularities in the wall. He proposes a parameter:

$$E/S$$

where E = perimeter
S = cross-sectional area

Which is $1/a$, the reciprocal of radius, for a perfectly cylindrical tube, but will increase if fissures parallel to the tube axis are present. Corrugations normal to these fissures cannot so easily be taken into account but they will be more liable to cause irregular gas motion. The proposed parameter would, for example, replace $1/a$ in equation (3.1).

Turbulence has been shown to be present in the "steady" air flow in brass instruments (Elliott 1979). Whether turbulence exists in an alternating flow is not easily proved or described, but it will be aggravated by irregular walls both in terms of roughness and larger discontinuities. Backus (1964C) shows that in the clarinet the Q's of resonances drop with increasing steady flow, though there are major discontinuities provided by the fingerholes which are not present in brass instruments. Benade (1959) shows higher attenuations in plastic tubes with fingerholes than for similar tubes without.

Turbulence effects are not easily formulated, and simple attempts (e.g. Binder 1943) have not progressed beyond empiricism. More rigorous analyses (e.g. Howe 1979) are highly complex and still do not fully represent the phenomena. Although an analysis of turbulence is beyond the scope of this thesis, it must be recognised that this could be a source of energy loss and also cause measurement errors.

3.2.3 Radiation and other losses

The theory for radiation impedance of an open end of a cylindrical tube is well documented (e.g. Kinsler and Frey 1962). For long narrow tubes this loss is relatively small and has little effect on impedance prediction comparing a theoretical open end and a "perfect" open end (i.e. reflection coefficient at open end = $-1+j0$). However, the model used for radiation impedance critically affects the prediction of transfer function, and discrepancies with measured data do arise. It will be shown in section 3.3.2 how the internal loss may be derived from measured data without knowledge of the radiation impedance, therefore an approximate radiation impedance is justified for comparison purposes only.

Radiation from brass instrument bells is a rather more difficult problem. Elliott (1979) has justified the use of the radiation impedance of a pulsating hemisphere and used it with some, though not complete, success. Due to its larger radius, the bell will make little contribution to the loss mechanisms discussed above compared to the narrower portions of the instrument, but will provide a higher radiation loss than for the case of the open ended cylindrical tube. This case is discussed further in section 3.4.

There are numerous other possible loss mechanisms which need to be acknowledged. Benade and Gans (1968) quote two more other than those discussed above; vocal tract loss and reed loss. For brass instruments these are properties of the player and not the instrument, so they are not the concern of this thesis.

Wall vibrations are another source of energy loss. Fay (1940), for his measurements on cylindrical tubes, needed to bury his experimental tube in sand as "without some such precaution, conditions in the sound field are unstable". Backus and Hundley (1966) showed with a simple theory that wall vibration due to the pulsating air column in organ pipes is insignificant. This subject applied to brass instruments is investigated in Chapter 4.

A further source of loss in brass instruments is due to leaks, not just arising from wear and damage but also from badly made instruments e.g. loosely fitting slides and valves, sub-standard water keys and badly soldered joints may all contribute to leaks. These effects are impossible to quantify and will not be discussed further in this thesis.

3.2.4 Prediction of input impedance and transfer function

Prediction of input impedance and transfer function for a cylindrical tube can be made by evaluating equations (B7) and (B8).

To do this, values for γ, Z_c and Z_ℓ must be chosen.

Values for γ and Z_c can be calculated from either equations (3.6) and (3.7) or equations (3.8), (3.9) and (3.10).

The termination impedance is evaluated using Nederveen's (1969) formulation:

$$Z_\ell = \frac{\beta a^2 |Z_c|}{4} + j0{\cdot}61\beta a|Z_c| \qquad (3.11)$$

This may be considered a reasonable approximation. The constant "0·61" is theoretically derived for an unflanged tube termination (Levine and Schwinger 1948).

Examples of impedance and transfer functions calculated using the data plotted in figures (3.1) and (3.2), are plotted in figures (3.3) and (3.4).

The difference in heights of the impedance maxima is mainly due to the approximation of attenuation, and at low frequency partially due to the approximation of the characteristic impedance. The difference between the positions of the maxima is due to the approximation of the wave number, and is by far the most significant difference.

3.2.5 Summary

The principal contributions to acoustic energy loss in brass instruments have been reviewed, discussed and evaluated for a simple case. It has been shown that the theory for viscous and thermal losses is well established and convenient equations are quoted. Surface roughness and turbulence are found to be difficult to formulate, but are also considered major contributory factors when

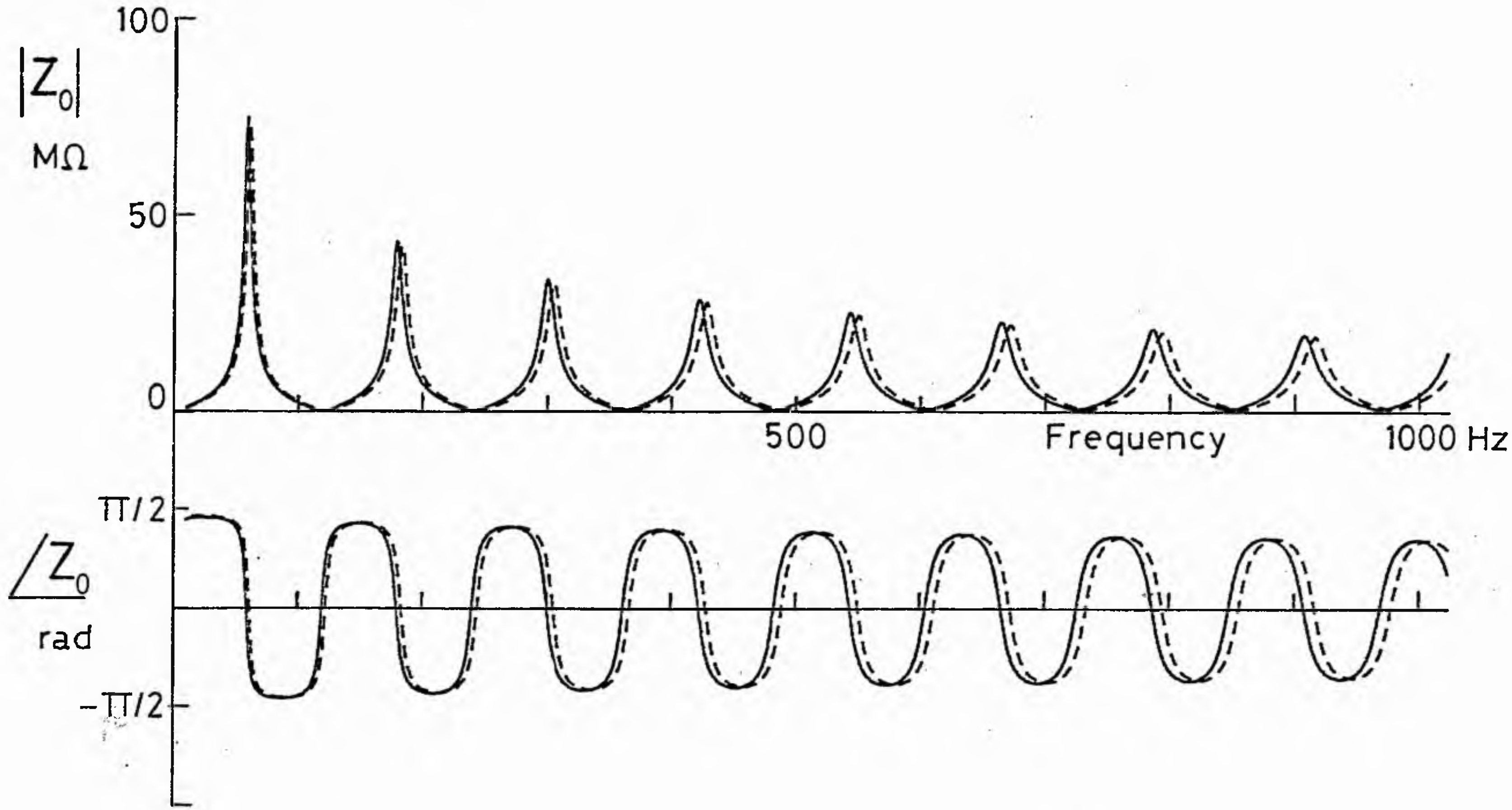


Figure 3.3 Theoretical input impedance for a cylindrical tube radius 0.0055 m and length 1.4 m "Exact" (solid) and approximate (dashed).

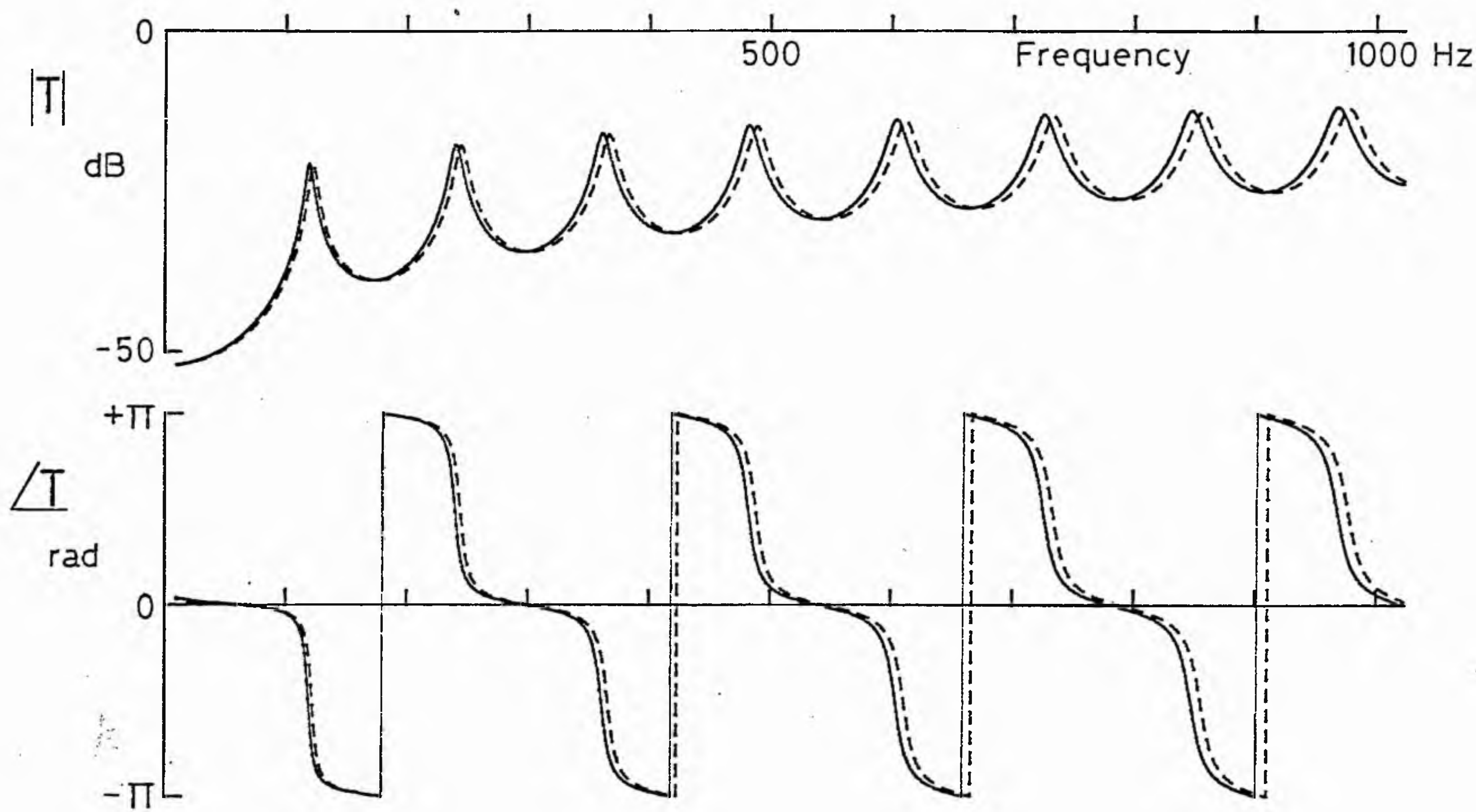


Figure 3.4 Theoretical transfer function for a cylindrical tube radius 0.0055 m and length 1.4 m "Exact" (solid) and approximate (dashed).

experimental results do not correlate with theoretical predictions. Other loss mechanisms are acknowledged. The subject has previously been reviewed (Benade 1959, Benade and Gans 1968) but results are only presented for "rough measurements" and ostensibly intuitive estimates.

The discussions of this section combined with practical considerations have led to a series of measurements on cylindrical tubes in various physical conditions and also on portions of brass instruments to investigate the effect of the quality of internal surface finish.

3.3 Measurement of Losses

3.3.1 Measurement of impedance and transfer function

The methods used to measure impedance and transfer function are the same as those of Elliott (1979) with minor modifications. The methods are briefly reviewed below and a detailed account can be found in the above reference.

The system to be measured is acoustically excited by an 18" loudspeaker (Cetec 8840) enclosed in a large box and coupled to the test-piece by a horn. The excitation signal (frequency f) originates from a programmable oscillator (Adret type Codasyn 201) and is divided to produce a sine wave of f/480 and a pulse train at f/20 to trigger data sampling at 24 samples/cycle. The signal at f/480 is fed through a programmable attenuator (Physics Department, University of Surrey) and then amplified (HH type TPA100-D) to drive the speaker.

To measure impedance, a plane of measurement must be defined, and the pressure and volume velocity signals measured there. The anemometry system used is a constant temperature hot wire system (Disa type 55K), which is calibrated before each experimental run using a set of flows previously determined with a Pitot tube and an inclined tube manometer. A computer program then performs a least squares fit of the calibration data to a suitable power law (program PKALO: see appendix D), such that the linearisation of the velocity signal can be performed by the computer during an experimental run.

The pressure signal is detected with a horn coupled probe microphone (B & K type 4170) and amplified (B & K type 2608). A calibration file to compensate for the frequency response of the microphone and the non-uniform profile of velocity across the measuring plane is stored on the computer and is used to scale the measurements.

To measure transfer function the internal pressure signal is the same as that for the impedance measurement. The external pressure signal is measured at some defined plane dependent on the experimental testpiece and purpose. The type of microphone used is similarly dependent and suitable calibration files have to be created for individual sets of transducers. For the measurements in this thesis the microphone used was a B & K type 4134 fitted with a probe 0.24 m long and 0.001 m diameter.

The computer used to control the experiments is a Data General NOVA 4 minicomputer and data is sampled by a two channel analogue input/output system (Micro Consultants type VHS modular 15).

The modification to the system of Elliott (1979) is that impedence and transfer function can be measured almost simultaneously by feeding the internal pressure signal to one channel of the ADC and having the facility to select either the velocity or the external pressure signals to the other channel. A program which can handle all the switching, calibration files, sampling and data was written (PZTRUNO). The need for this is due to the temperature drift during and between separate measurements of impedance and transfer function which cause a misalignment of data which adversely affects the results produced by the methods described in section 3.3.2.

For measurements on cylindrical tubes the planes of measurement are easily defined. The plane of measurement of impedance is simply a cross-section of that tube at a particular distance from the open end. The plane of measurement for the external pressure signal is defined to be in the plane of the open end.

To keep within the dynamic range of the anemometer (the most limited of all the equipment used) the computer program controls the level of excitation by controlling the programmable attenuator. The constant value of volume velocity which it attempts to maintain is preset at the start of a run and is quite critical due to the relatively large cross-section of the tube compared to the cross-sections for which the apparatus was designed (viz. trombone mouthpiece throats).

At impedance maxima where the volume velocity is small compared to the pressure the system has to drive very hard to maintain the preset level, yet if the level was set lower the signal to noise ratio of the anemometer signal would become unnacceptably low. An optimum level was found to be 12 cc/s.

The other critical parameter is the anemometer bias air flow. The hot wire system cannot measure direction of flow, and if just an alternating velocity was being detected, a rectified alternating signal would be produced. To overcome this a steady air flow is superimposed upon the alternating flow and as long as the air velocity of this bias flow is greater than the peak air velocity of the alternating flow rectification will not occur. The phase response of the anemometer is found to be critically dependent upon this bias flow up to a velocity of about 4 m/s; the lower the flow, the more deviant the response. If the flow is too high turbulence will occur and swamp the desired signals. For the cylindrical tube used here an optimum flow was found to be 2 m/s. This required only small corrections to the phase of the measurements (see section 3.3.2) and only gave turbulence problems in the frequency range 10 Hz to 100 Hz. The loss of data in this range is considered a reasonable price to pay for good data in the range 100 Hz to 1024 Hz. Measures of impedance and transfer function for a tube 1.402 m long and 0.0055 m radius are shown in figures (3.5) and (3.6) along with their theoretical counterparts.

3.3.2 Derivation of loss from measurements of Z_0 and T

The basic equation needed for this section is derived in appendix B and there referred to as (B10). For convenience it is repeated below:

$$R_0.e^{2\gamma\ell} - T(1 + R_0).e^{\gamma\ell} + 1 = 0 \tag{3.12}$$

Of the parameters in the equation: T (transfer function) is measured, R_0 (input reflection coefficient) is calculated from input impedance (measured) and characteristic impedance, ℓ (length) is measured, leaving γ (propagation coefficient) to be calculated. This is simply done by solving the quadratic in $e^{\gamma\ell}$, taking the natural logarithm and dividing by the tube length.

The input reflection coefficient is defined as the ratio of the reflected pressure wave to the incident pressure wave (Connor 1972):

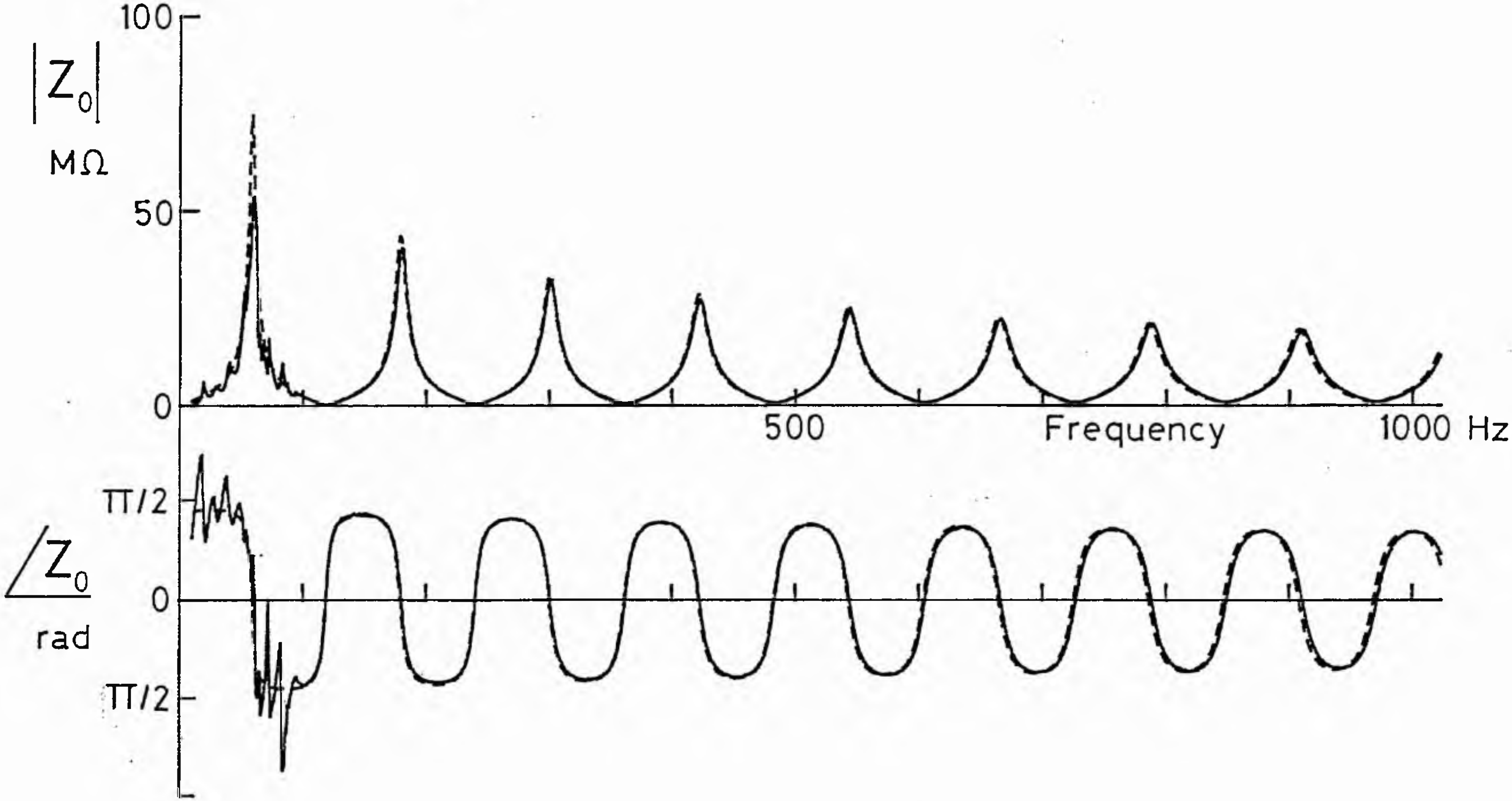


Figure 3.5 Input impedance for a cylindrical tube radius 0.0055 m and length 1.4 m Experimental (solid) and theoretical (dashed).

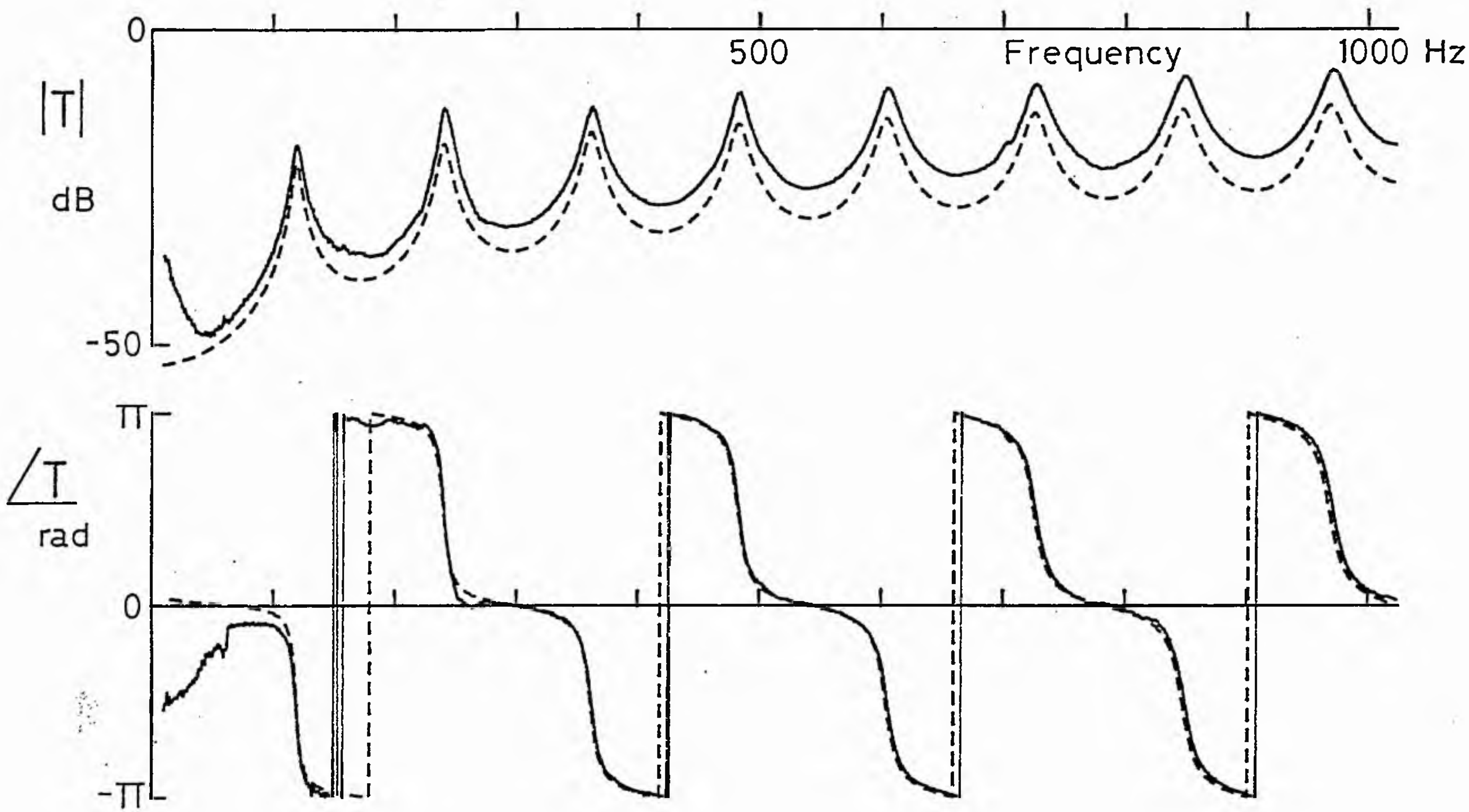


Figure 3.6 Transfer function for a cylindrical tube radius 0.0055 m and length 1.4 m Experimental (solid) and theoretical (dashed).

$$R_0 = \frac{Be^{\gamma \ell}}{Ae^{-\gamma \ell}} \tag{3.13}$$

where A and B are constants.

If the termination impedance is a smooth function of frequency and the losses in the tube are smooth functions of frequency, then it is a corollary of equation (3.13) that the magnitude of the input reflection coefficient is also a smooth function of frequency. These conditions are satisfied for the theoretical data as in figures (3.1) and (3.2) (termination impedance is calculated from this data also) and a theoretical input reflection coefficient for a cylindrical tube is plotted in figure (3.7). The smoothness of this parameter as a function of frequency is a sensitive test of the data; small random inaccuracies show up as "noise" and more general deviations as "peaks and dips". In this case R_0 is calculated from the equation (Connor 1972):

$$R_0 = \frac{Z_0 - Z_c}{Z_0 + Z_c} \tag{3.14}$$

which is simply derived from equation (3.13). We therefore have a good criterion with which to judge the quality of correction for the anemometer phase characteristic and the quality of choice of characteristic impedance; the two major factors contributing to poor and inconsistent results.

It has been shown that if the values of such impedances are plotted on an Argand diagram the result is a series of circles, one for each impedance peak, the centres of which lie on the real axis (Pratt and Bowsher 1979). A computer program (SQPDRAW) was written (Munro 1979) to fit circles to the measured impedance data near each

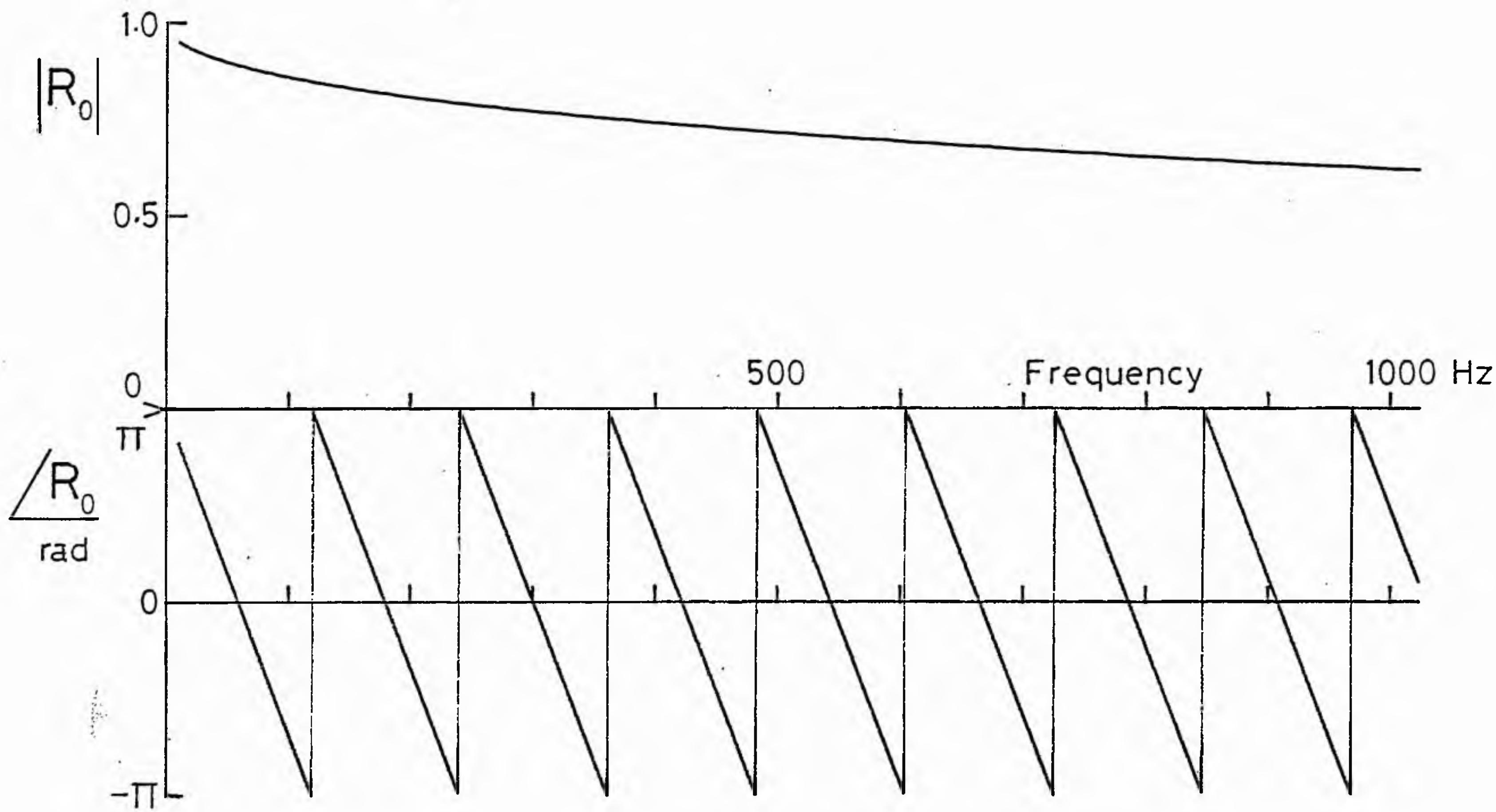


Figure 3.7 Theoretical input reflection coefficient for a cylindrical tube 0.0055 m radius, 1.4 m length.

peak and thus the centres deduced. By performing such a series of fits one can derive a series of phase corrections needed to fulfil the requirement of a zero imaginary part at a maximum of the real part of impedance. A computer program was written (POGGART) to take the data output from SQPDRAW, interpolate between the points (one for each peak), and adjust the impedance data to fit the criterion of zero phase coinciding with a maximum real part (i.e. zero imaginary part).

An example of a measured tube impedance before and after phase correction is shown in figure (3.8). The correction is small (about 0.1 rad. at 1000 Hz) but significant to the application of equations (3.12) and (3.14).

To derive an approximate value of characteristic impedance the property of equation (B7) that log (Z_0/Z_c) is "symmetrical" about 1.0 is used. A computer program (PIT) iterates through a cycle of trying a value for Z_c, testing the criterion by finding the average over all values of $\log_{10}(Z_0/Z_c)$ and adjusting Z_c accordingly until the average is tolerably near 1.0. Values generated by this method using the cylindrical tube data fall in the range 4.45-4.75 MΩ which is gratifyingly small. Different values of Z_c are expected for tubes of different wall loss characteristics due to the definition in equation (3.6). Figure (3.2) shows that the "exact" theory gives a characteristic impedance which has a slight frequency dependence. It is clearly impracticable to use this subtlety as one cannot expect the characteristic to be the same for tubes with different loss (i.e. the theoretical values cannot be used with experimental data) and there is no method as yet of deriving this frequency dependence from the experimental data.

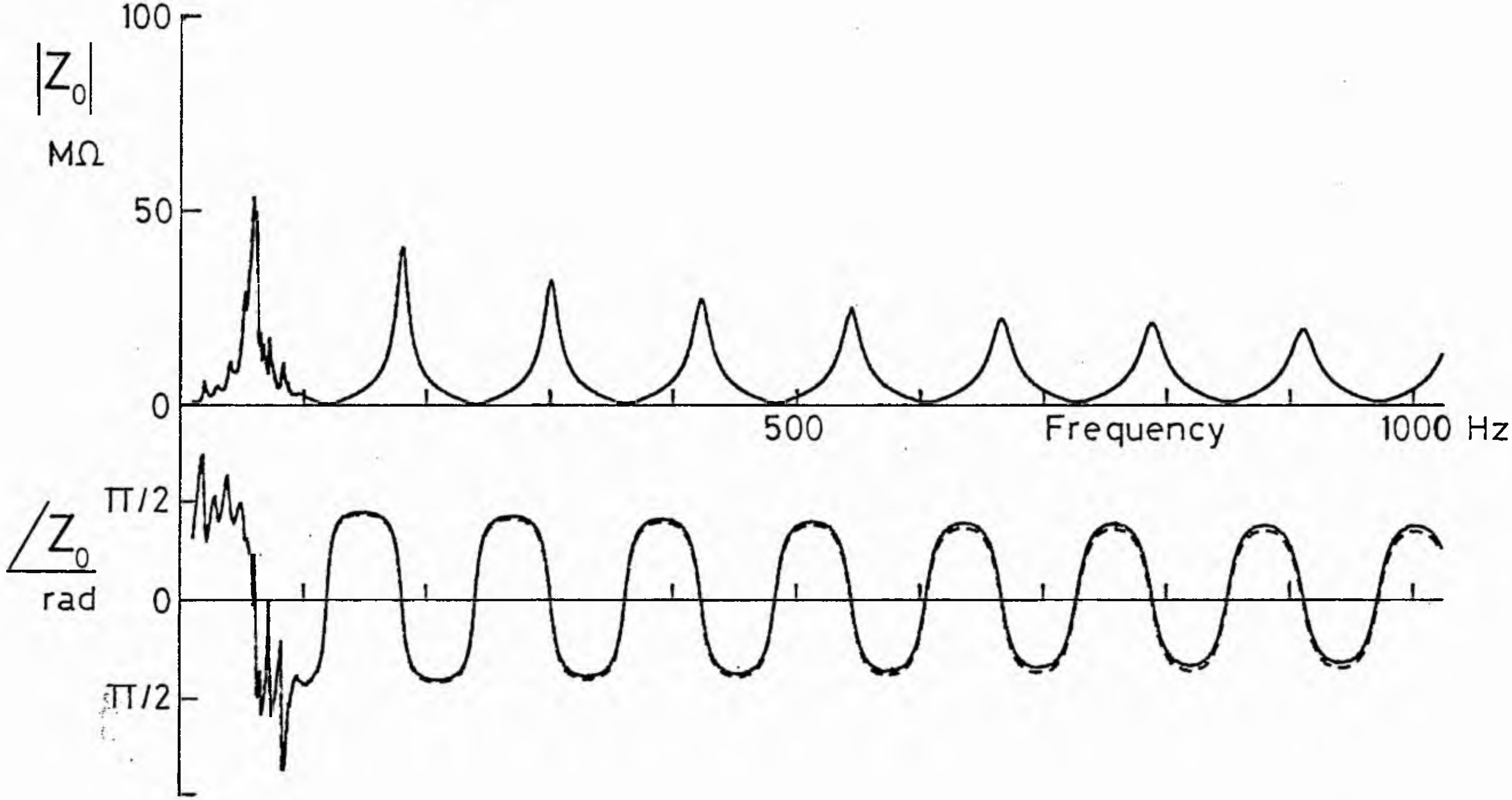


Figure 3.8 Experimental input impedance for a cylindrical tube before (solid) and after (dashed) phase correction.

The length of the tube is simply its geometrical length between the plane of impedance measurement and the plane of the open end of the tube. "Effective lengths" which are so often used are for "rule of thumb" calculations to take into account the deviations from the ideal case caused by the finite termination impedance and to some extent the internal losses discussed in this chapter. In the derivation of equation (3.12) the termination impedance as a parameter disappears because it is implied in the measurements and therefore does not have to be accounted for.

Equation (3.12) is solved by computer (PWATERQ), a sample solution for theoretical data is shown in figure (3.9). There are two solutions, either of which may be correct depending on the frequency range. The stepped nature of the imaginary part (wave number) is due to taking the logarithm of a complex number, the imaginary part being an angle which, on the computer, is limited to the range $\pm \pi$. At low frequency it is obvious which solution is correct (that with the positive imaginary part), following this solution with increasing frequency the solutions "swap" at about 60 Hz and an increment of $2\pi/\ell$ is needed at about 115 Hz etc. The "correct" solution corresponds to the lower locus of the real part (attenuation coefficient). It is the attenuation coefficient we are particularly interested in and the difference between the two solutions of this is small, even at high frequency.

The methods discussed in this section can now be applied to experimental tubes of different physical states. The theoretical result in figure (3.9) is used as a yardstick for all the experiment based results.

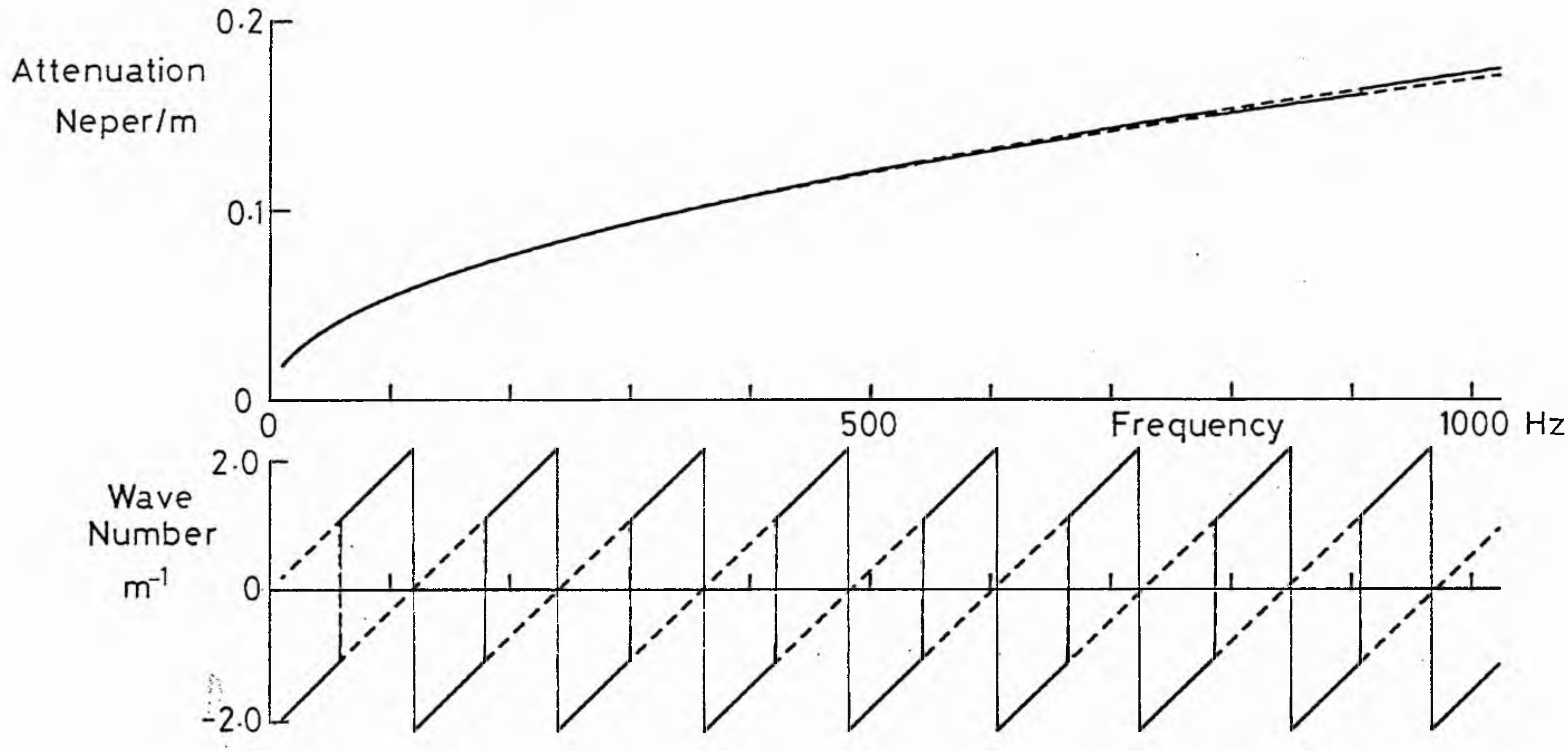


Figure 3.9 Theoretical propagation coefficient derived by equation (3.12) from the data in figures 3.3 and 3.4. Solution 1 (solid) and solution 2 (dashed).

3.3.3 Results and discussion

The experimental testpiece was a tube 1.402 m long and 0.0055 m radius. The tube was made of brass and had a wall thickness of 0.00165 m. The tube was measured in six different states thought representative of some conditions found in tubes used in brass instruments. The attenuation coefficients calculated from the six impedance and transfer function pairs are plotted in figures (3.10) to (3.15). The theoretical curve (as in figure (3.9)) is plotted on all these as well for comparison.

Figure (3.10) shows the result for the tube after it has been thoroughly cleaned to a smooth and shining internal finish. The mild undulations of the curve are most probably due to the approximations discussed in section 3.3.2 i.e. choice of characteristic impedance and the phase correction for the anemometer. The smaller deviations are due either to noise in the measurements or non-linearities in the system e.g. turbulence or circulation currents. Despite these irregularities it can be seen that the measured attenuation is, on average, of slightly greater value than that predicted by the theory.

Figure (3.11) shows the attenuation curve for the tube as found in the laboratory before cleaning. The picture is very similar to that in figure (3.10) and is not a surprising result as the tube was quite clean anyway. It gives an indication of the good repeatability of the measurements considering the noise and undulations disguising the true curve.

A traditional "primer" for brass instruments is a mixture of brandy and honey. Its powers are reputed, for example, to speed up the "blowing in" process (analogous to "running in" a car), and to

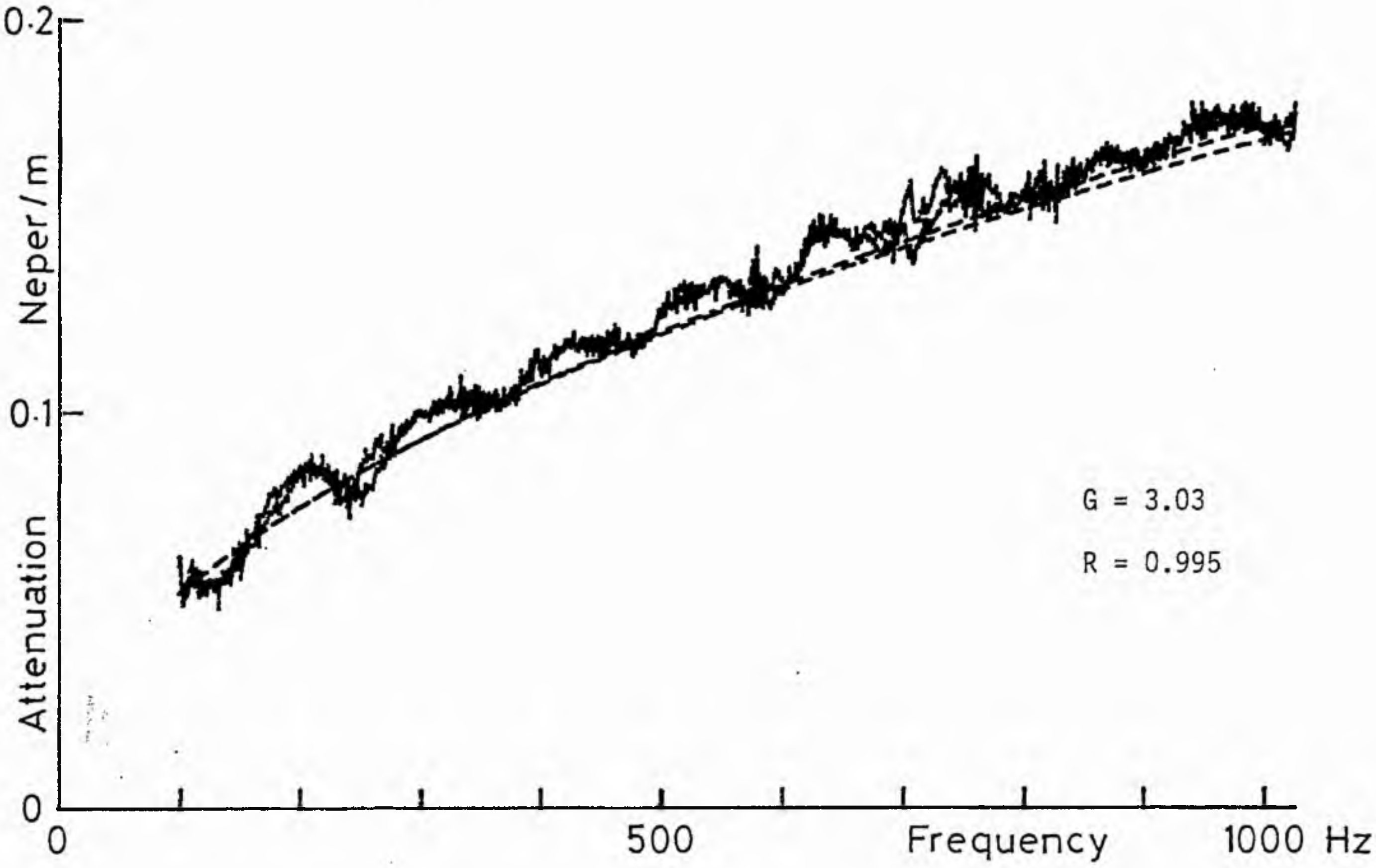


Figure 3.10 Attenuation coefficient for the clean cylindrical tube (solid) theoretical attenuation as in figure 3.9 (dashed).

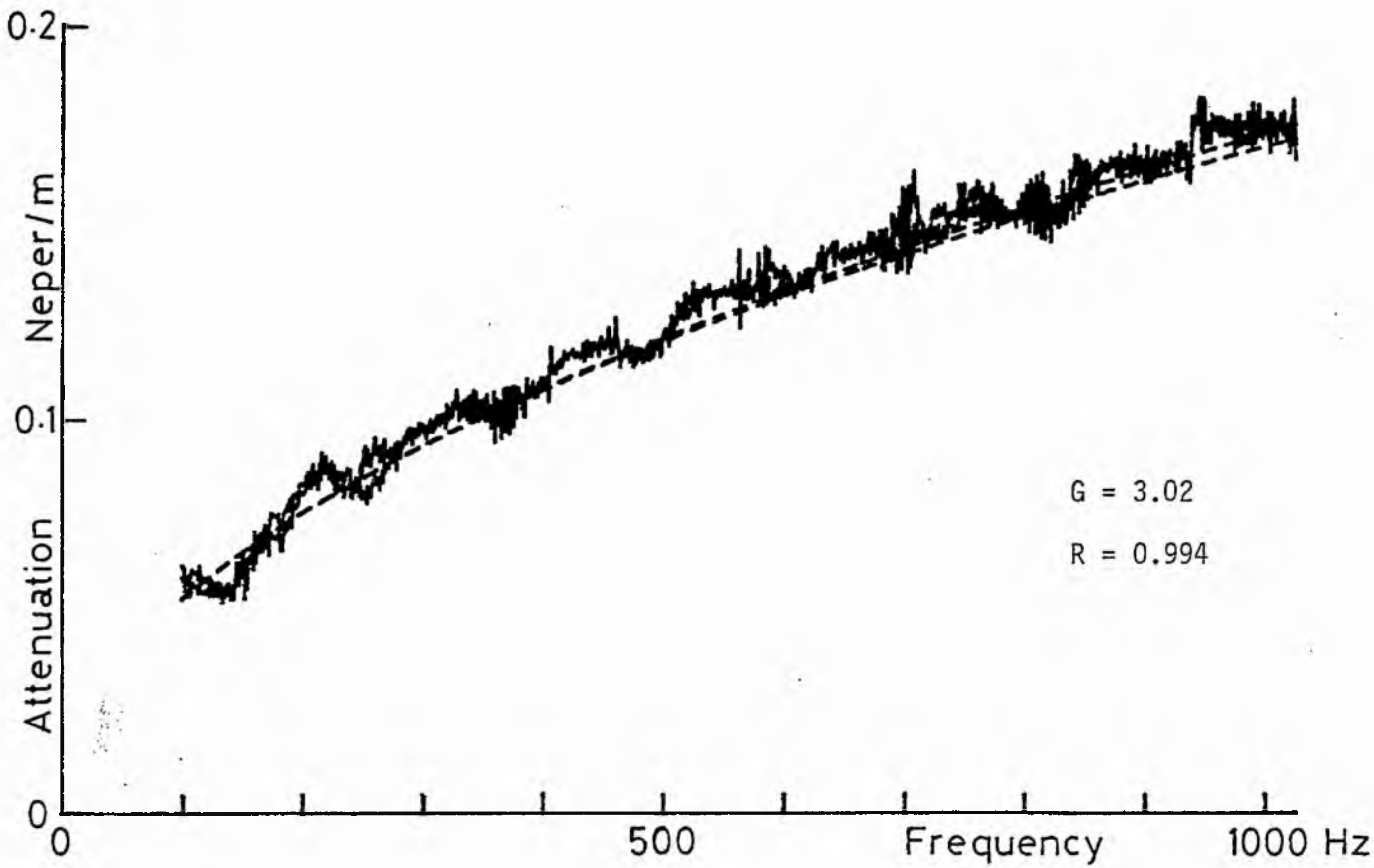


Figure 3.11 Attenuation coefficient for the tube as found in the laboratory.

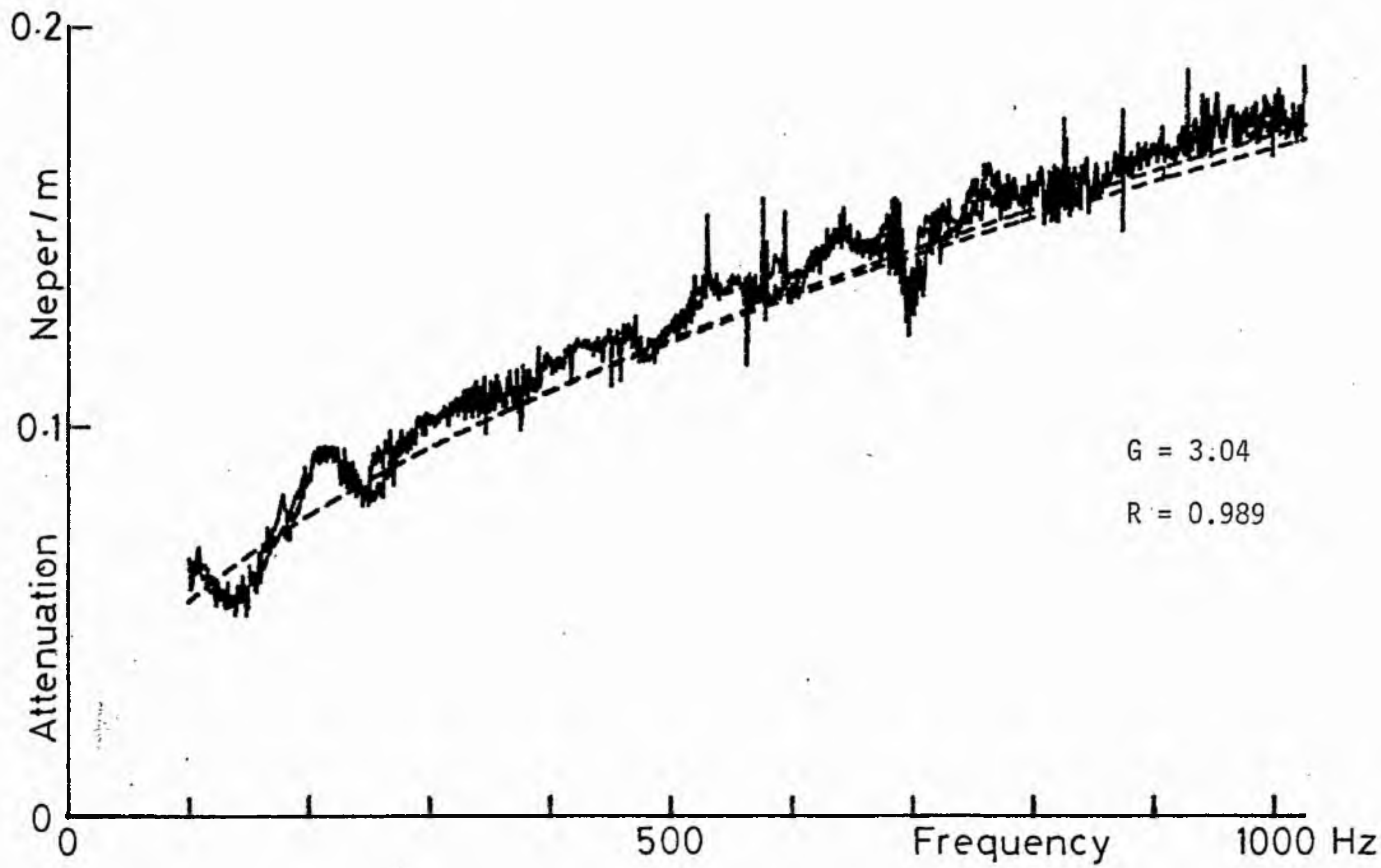
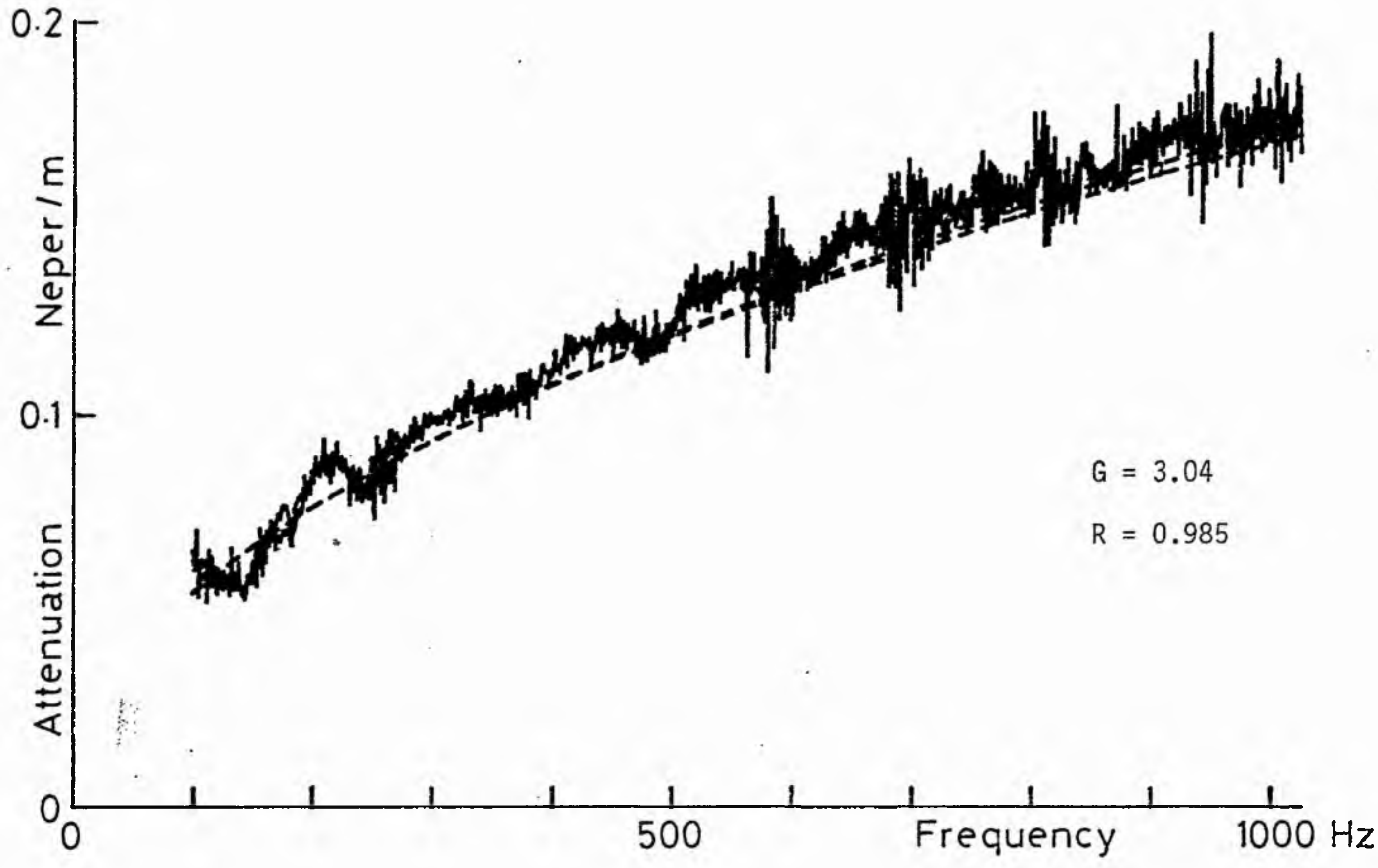


Figure 3.12 Attenuation coefficient for the tube internally coated with a mixture of brandy and honey.



<u>Figure 3.13</u> Attenuation coefficient for the tube with the internal surface roughened.

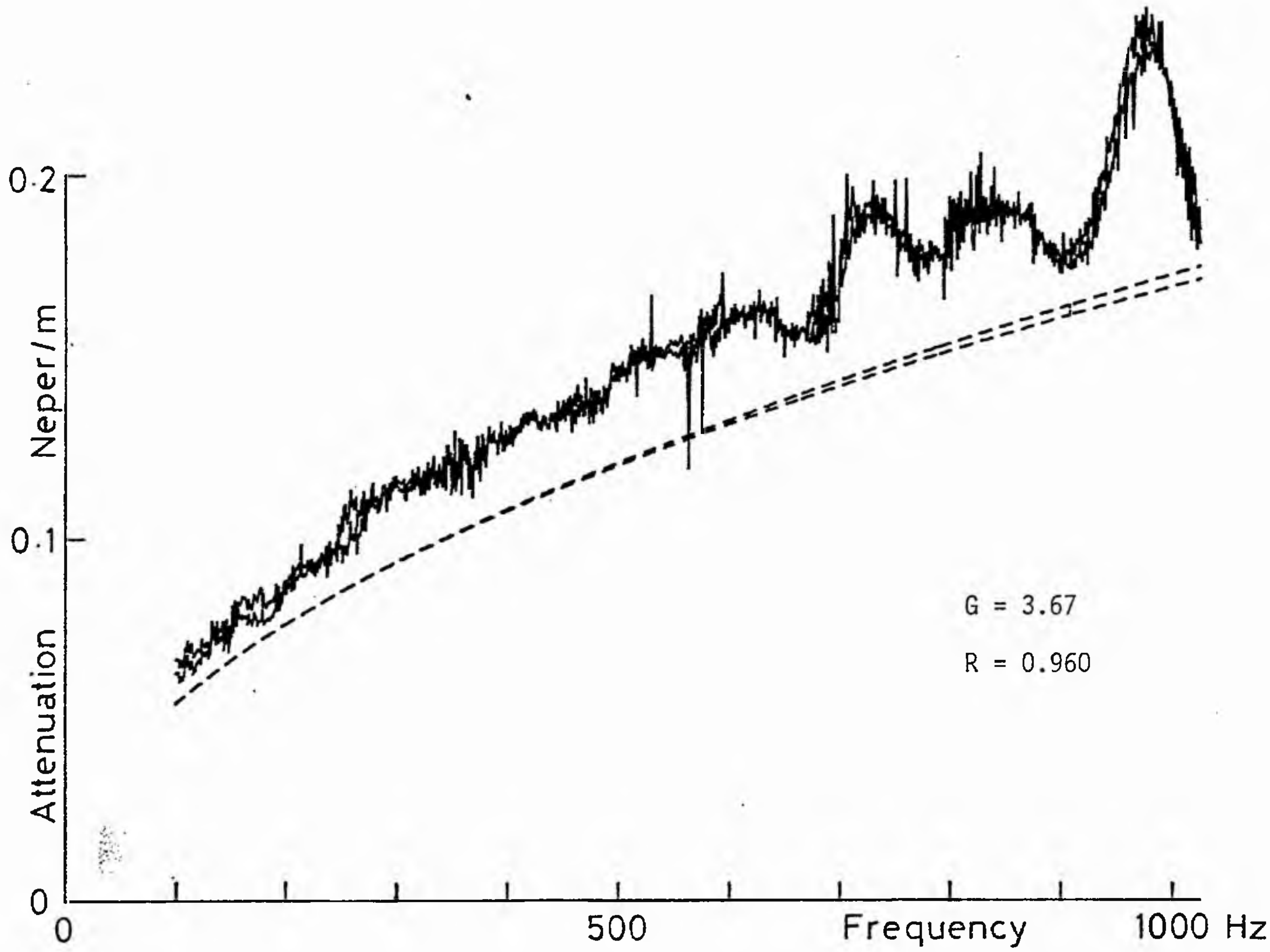


Figure 3.14 Attenuation coefficient for the tube internally coated with a coarse peanut paste.

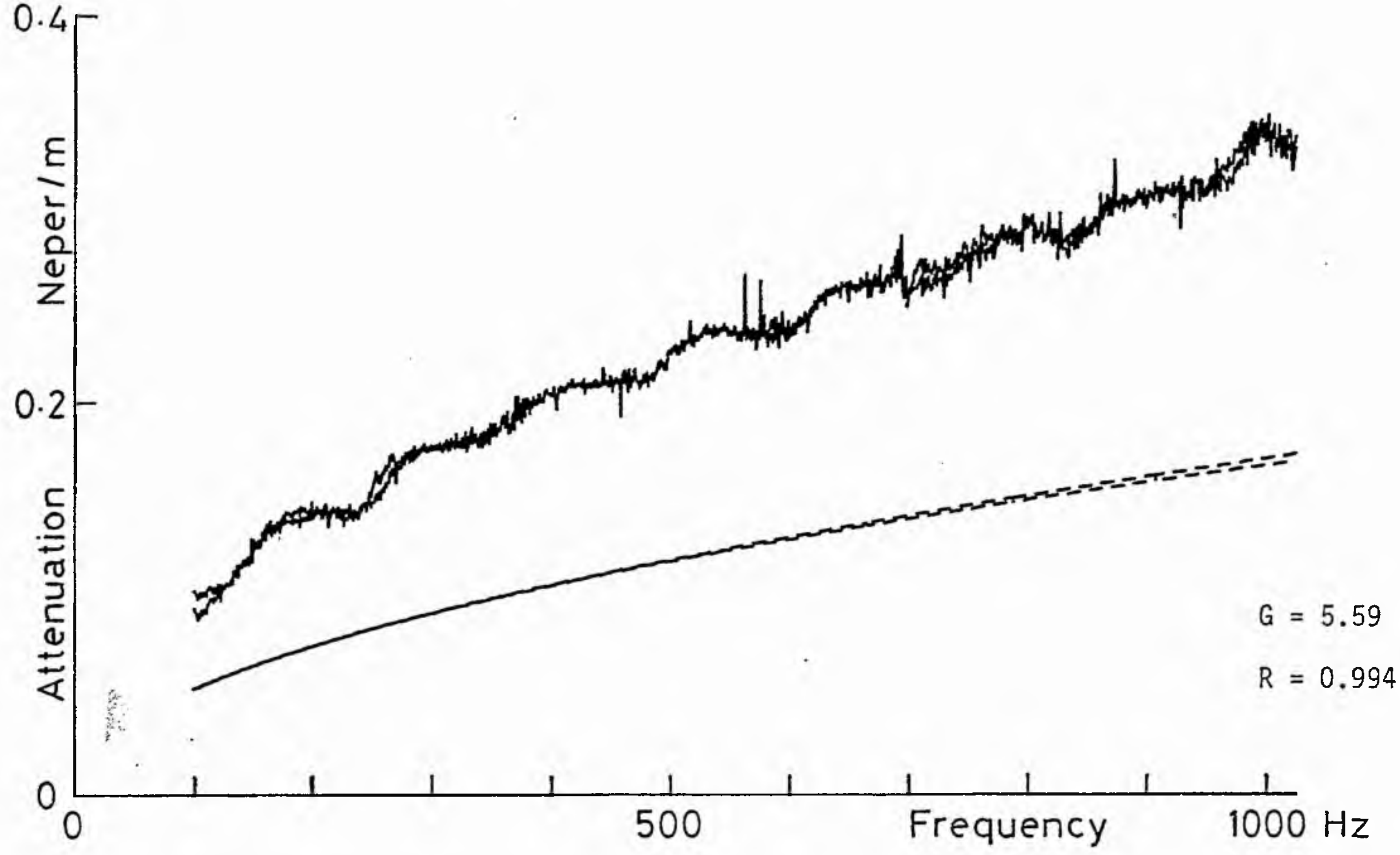


Figure 3.15 Attenuation coefficient for the tube with a strip of paper towel inserted along the length.

improve the "response" of the instrument. To test the effect of this upon attenuation the tube was coated internally with this potion and measured. The results are shown in figure (3.12). The surface produced by the coating was even and smooth, and thus would not be expected to produce any irregularities. However, the coating will reduce the internal radius of the tube and thus we expect an increase in attenuation. Inspection of the results shows this to be the case, albeit a small effect, though there is an unexplained irregularity at about 700 Hz.

The tubing used in brass instruments will show a variety of factory internal finishes. Some tubing is used in the condition it enters the factory, some tubing is drawn through or over a mandrel, some tubing is forced into shape by the passage of lead balls, some tubing is hydraulically "blown" into shape (see appendix C). To test the effect of a rough internal surface, the bore of the tube was roughened by the passage of numerous pieces of steel wire attached to a rotating rod, the ends of the steel wire being bent to present the cut face of the wire to the internal surface of the tube. This method successfully produced a heavily scored finish in the tube without removing much brass and thus not significantly increasing the bore diameter. The result of the measurements is shown in figure (3.13). Disregarding the noise and undulations, the result is very similar to that of the clean and laboratory condition tubes. The only distinction is a higher noise level which can be expected from rough walls causing unpredictable irregular motion of the air, though not enough to cause an overall greater loss of sound energy.

If an instrument is played often and not cleaned for a few years, or even months, there is a build up of deposit in the instrument consisting of food particles and other solids which issue from the player's mouth during playing, and also soluble substances which are left after moisture, which has condensed or run into the instrument during playing, has evaporated. To simulate this, peanuts were coarsely ground with water to form a paste containing a variety of size peanut "grains". This mixture was ladled into the tube and spread along the length using compressed air. The tube was then left overnight with air flowing through at low velocity to dry out. The tube was then measured and the results are shown in figure (3.14). The result shows a large increase in attenuation compared to the previous cases. The effect of this peanut lining will be to restrict the bore, thus implying an increase in attenuation, and also to provide a very irregular internal surface. The noise on these results is comparable to that for the tube coated with brandy and honey, but the undulations are large from about 600 Hz upwards which is indicative of a breakdown in the theory. This is not surprising since the theory is for an idealised cylindrical tube, whereas this is a case where, internally, the tube is now far from cylindrical.

As a test of the technique a further measurement was made. With the tube in its clean condition, a strip of paper towel 1 cm wide was placed inside the full length of the tube. Its attenuation-frequency curve was determined and is plotted in figure (3.15). Note that the attenuation scale is different from that of figures (3.10) to (3.14), though the dashed line is the same as for figures (3.10) to (3.14). As expected, the attenuation is much higher than that predicted by theory, and provides a comparison to the previous five cases. Noise and undulation levels are comparable to those of the clean, laboratory condition and brandy and honey coated tubes.

As a further comparison, the results were fitted to equation (3.1), and thus a value of G for each case deduced. These values and their correlation coefficients are presented on their corresponding data plots in figures (3.10) to (3.15).

The above results are indicative of the increased attenuations caused by various physical situations found in brass instrument tubing. It would be advantageous to be able to make similar measurements on complete or parts of instruments which could be used in conjunction with subjective testing to investigate if and how the various values of attenuation can be detected by a player, and thus aid the brass instrument designer. The next section discusses some possible extensions of the theory and presents some results for measurements on instruments.

3.4 Losses in Real Instruments

3.4.1 Applicability of measuring technique

The equations used for the method described in section 3.3 for determining attenuation do not directly apply to brass instruments. The solutions of the transmission line equations quoted in appendix B (sections B1 and B2) are only valid for the case where the distributed parameters (Z and Y) are constant along the length of the line. In the cylindrical tube analogy the distributed parameters depend upon the tube radius, and therefore in a brass instrument they vary in a mathematically arbitrary way along the length of the instrument.

The transmission line differential equation pertaining to this case is quoted in section B3 of appendix B, followed by the first approximate solution. This solution implies an "average" propagation coefficient for the tube and the limitation is to where the distributed parameters are slowly varying such that reflections due to their variation are insignificant.

There are generally two regions in a brass instrument where this condition is definitely not fulfilled: the backbore of the mouthpiece (see figure (1.2)) and the rapidly flaring portion at the bell end.

This solution also relates to the "harmonic mean radius" used by Benade (1959, et Jansson 1974) to estimate losses in instruments, and the limitations to the use of this solution relates to the fact that the bell flare is not included when his parameter is evaluated. The "harmonic mean radius" is used, for example, in equation (3.1). Jansson and Benade (1974) derive experimental values for attenuation in cylindrical tubes by using "peak to dip ratios" from measured impedance data and calculating from a conveniently approximate version of equation (B7). They also extend this method for similar measurements on real instruments by including most of the bell acoustics in the termination impedance; a technique which could be applicable to the method of section 3.3 as the termination impedance disappears in the algebra without assuming its form.

3.4.2 Results and discussion

A convenient first step when measuring real instruments is to measure trombone slides on their own. For most of its length a trombone slide is cylindrical; the flared sections being the backbore of the mouthpiece and the mouthpipe (a mildly flared section about 0.2 m long).

The measured impedance of a trombone slide (Boosey and Hawkes prototype) is shown in figure (3.16). The impedance is measured in the mouthpiece throat. For the loss derivation to apply the impedance must also fit the criteria discussed in section 3.3.2 i.e. the Argand diagram must be in the form of circles and the centres of these circles must lie on the real axis. These results can then be adjusted using a similar process to that employed when correcting for the anemometer phase response (the anemometer phase response does not need to be accounted for on measurements made in the mouthpiece throat because the bias velocity is high). On the assumption that the mouthpiece throat and backbore is the most significant contributory factor to the deviation from the circle centre criterion and that this section of the instrument is small enough to be modelled by purely reactive lumped acoustic components, the correction is made to the imaginary component of the impedance data (c.f. the phase component for the anemometer correction). The corrected data are shown in the dashed line of figure (3.16). It is easily shown that the transfer function data may be compatibly adjusted by the relation:

$$T_{corr} = \frac{T.Z_0}{Z_{corr}} \tag{3.15}$$

After this processing equation (3.12) may be solved and the result is shown in figure (3.17). Figure (3.18) is a result for similar measurements on another trombone slide (Sovereign large bore tenor). The lengths of the two slides are similar (1.59 m, 1.62 m respectively) though the B & H prototype's cylindrical bore is 0.127 m and the Sovereign's cylindrical bore is 0.139 m. This implies that the attenuation coefficient for the Sovereign's slide should be less than that of the B & H prototype's slide. Despite the large undulations it can be seen that this is generally so.

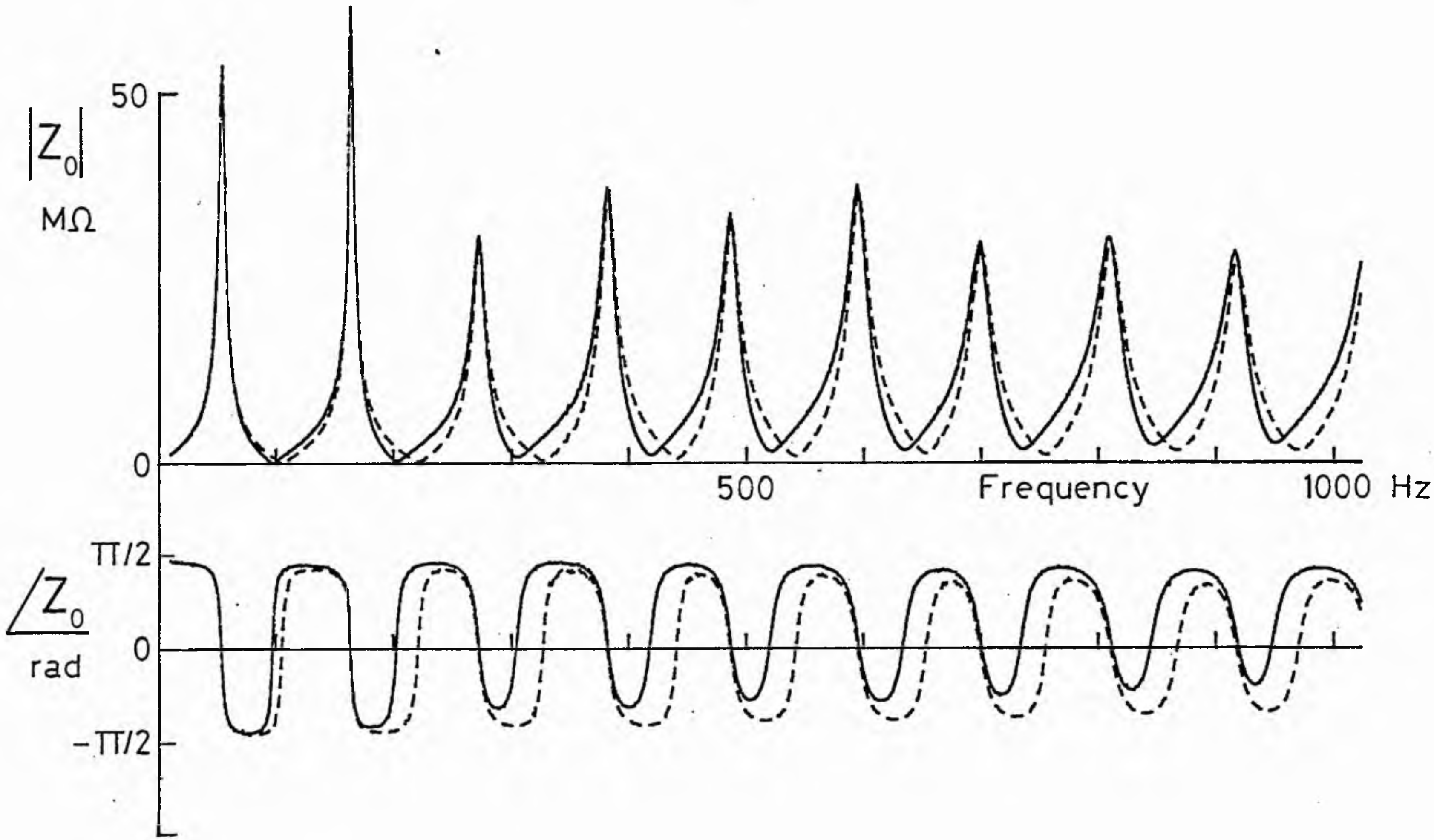


Figure 3.16 Input impedance of a trombone slide measured in the mouthpiece throat. Before (solid) and after (dashed).

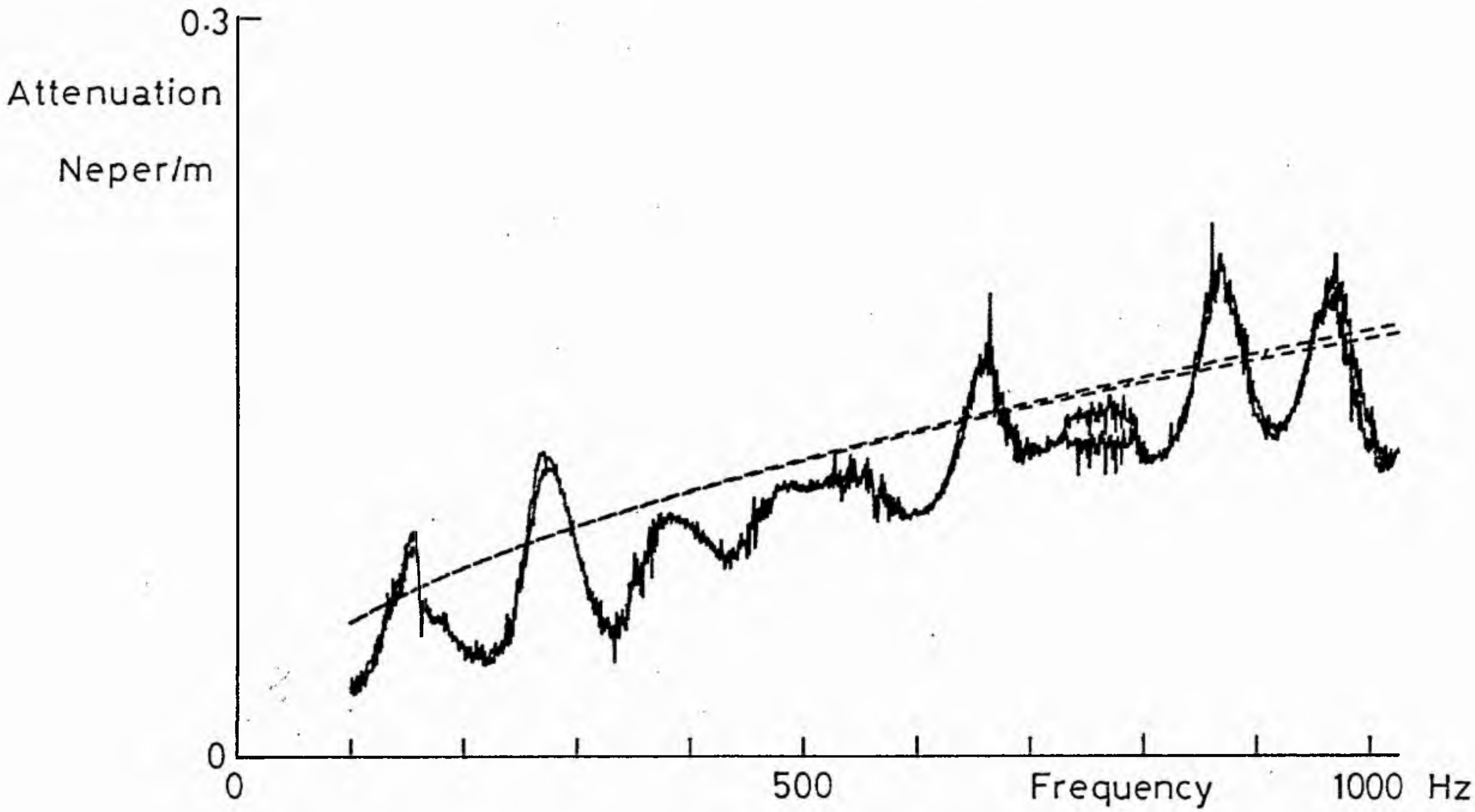


Figure 3.17 Attenuation coefficient for a medium bore trombone slide (solid). Attenuation coefficient as in figure 3.9 (dashed).

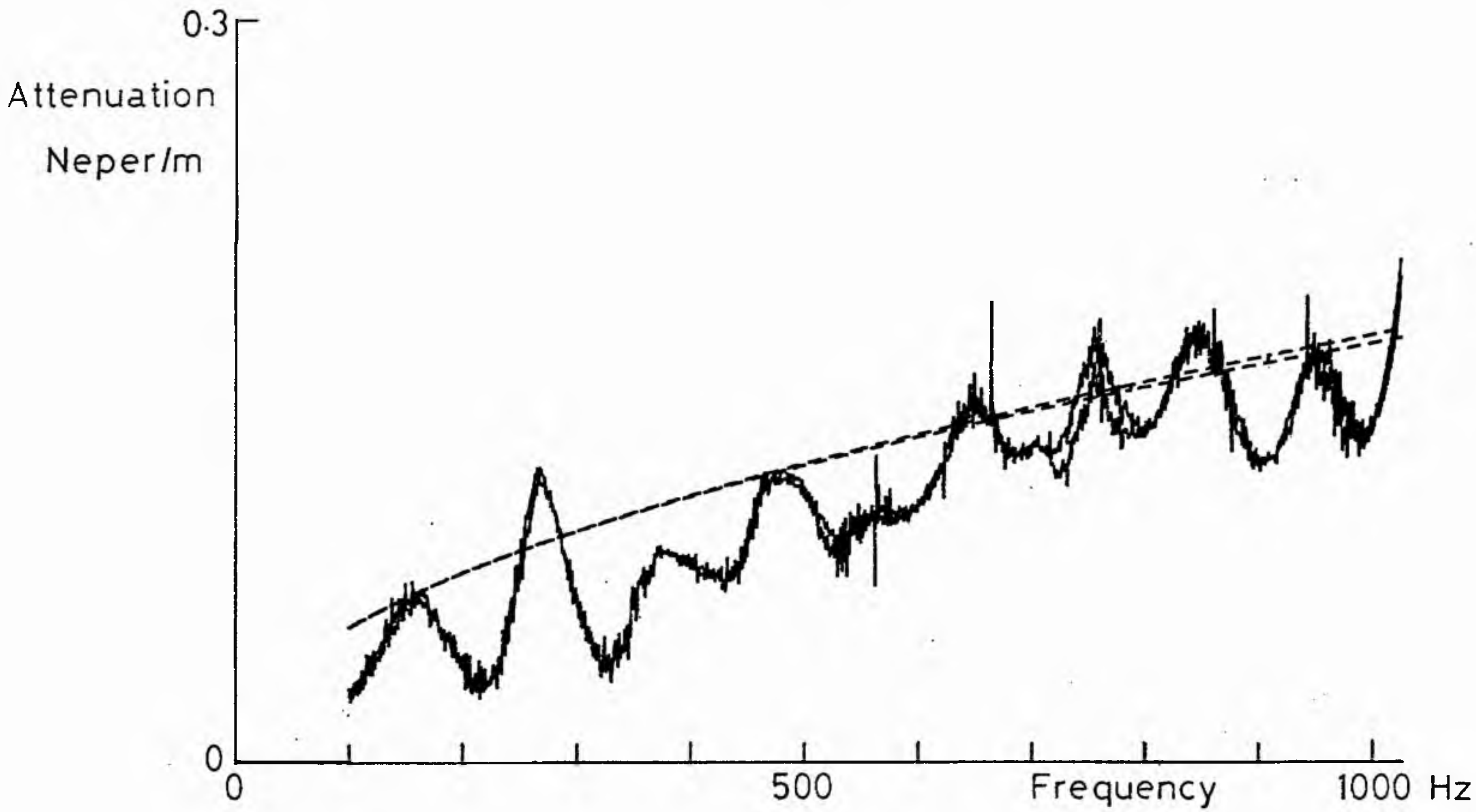


Figure 3.18 Attenuation coefficient for large bore trombone slide.

To take the extension further to include a complete instrument the bell must be accounted for. This can be done by the technique mentioned in section 3.4.1, i.e. include the bell in the termination impedance. This is achieved by measuring transfer function between the mouthpiece throat and the bell throat (see figures (1.1) and (1.2)). Results are processed as for the slide alone and are plotted in figures (3.19) and (3.20) which are respectively for the B & H prototype and the Sovereign. As can be seen, there is a complete breakdown of the applicability of the theory and nothing can be justifiably discerned from these results.

3.4.3 Summary

Measurements of impedance and transfer function for acoustic waveguides have been made using the system of Elliott (1979), and the limitations of applying this system to the testpieces in question have been discussed. It has been shown how it is possible to derive the transmission line propagation coefficient from these measurements, the real component of this coefficient being the parameter of interest. Measurements on cylindrical tubes in various internal physical states show the validity of the method and also indicate how these states affect attenuation. The cases studied are chosen to be representative of conditions found in brass instruments and show that the build up of deposit incurs a large increase in attenuation whereas mild internal roughness increases irregularities but not average attenuation levels. Measurements are made on trombone slides and complete trombones to test the limitations of the method finding that it is valid for very general conclusions on the slides but not applicable to complete instruments.

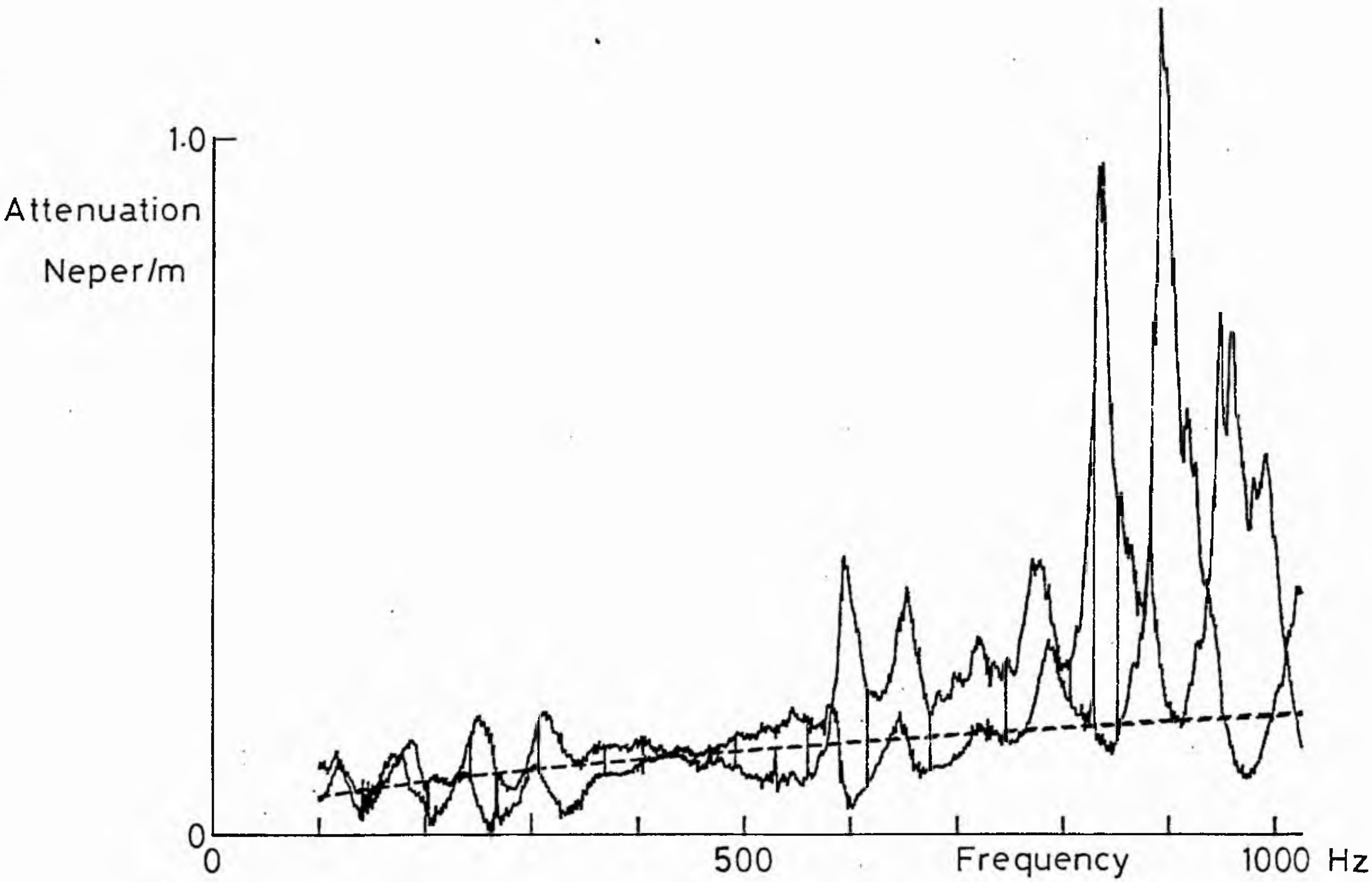


Figure 3.19 Attenuation coefficient for medium bore trombone.

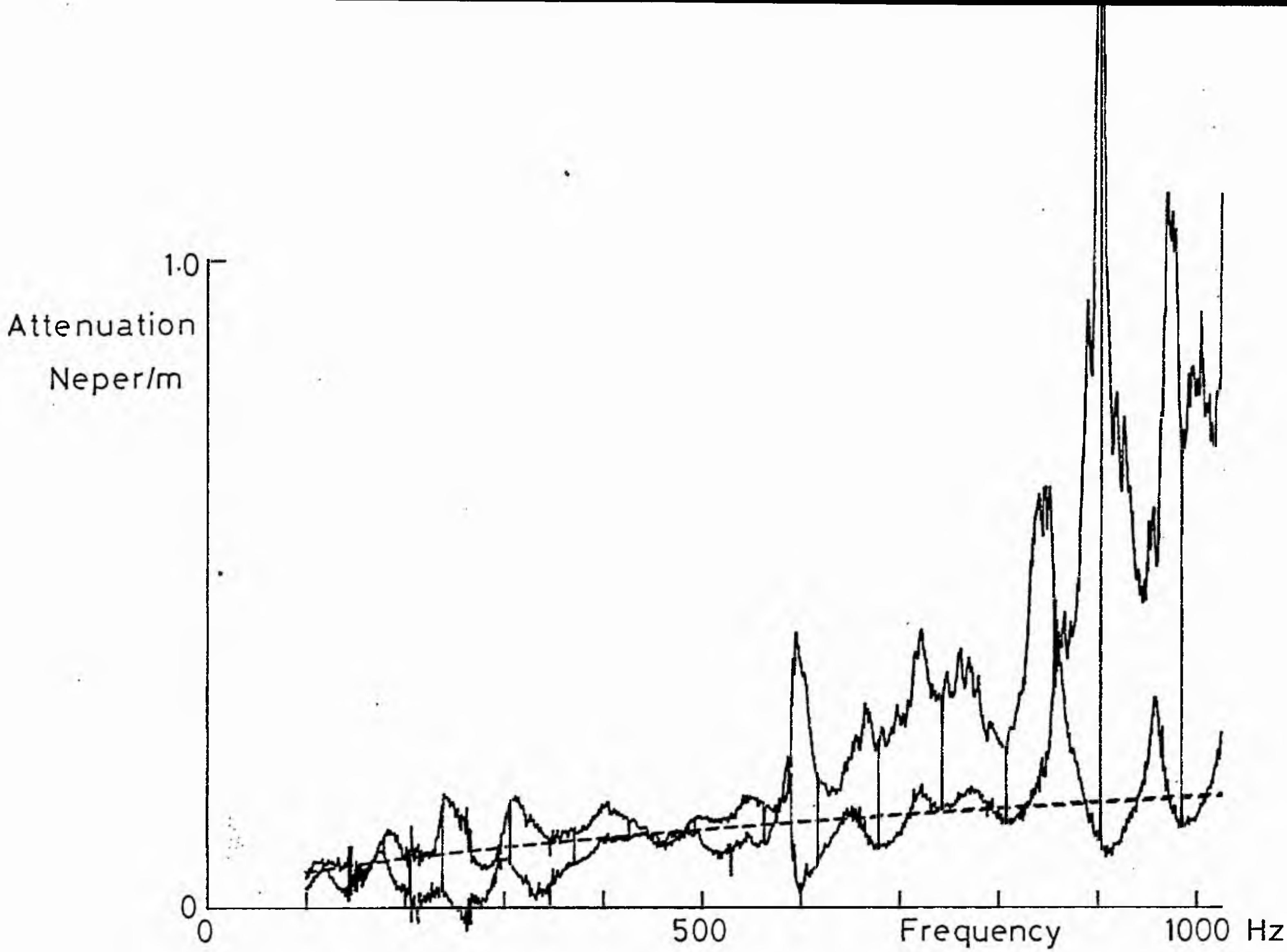


Figure 3.20 Attenuation coefficient for a large bore trombone.

CHAPTER 4 : VIBRATION PROPERTIES OF BRASS INSTRUMENT BELLS

4.1 Introduction

One of the many controversies surrounding the mechanics of brass instruments is the significance of the deviations from the ideal case of the instrument bell being a rigid boundary for the air column. Yielding walls can interact with the standing wave in the air column, internally dissipate energy and radiate sound from their outer surface: they can do all these whether considered yielding or resonant. If resonant at particular frequencies e.g. notes of the musical scale, will the effect be constructive, detrimental or insignificant?

The following sections report a preliminary study of the vibration properties of trombone and trumpet bells; mathematically intractable structures. Finite Element techniques and generalised vibration methods are used to generate natural mode frequencies and shapes, and to calculate the response of these modes when acoustically excited.

4.2 Finite Element Analysis

4.2.1 Formalisation

Although a comparatively new technique, the fundamental principles of finite element methods are well documented (e.g. Zienkiewicz 1971, Petyt 1977) and therefore only a brief description of the technique is given below.

The structure to be analysed is defined geometrically by a set of discrete points or nodes, each node being assigned appropriate degrees of fredom. A local region defined by surrounding nodes is an "element". The deformation properties of single element can be described by a simple set of displacements in terms of the degrees of freedom of the nodes defining that element. Thus, the deformations of the complete structure can be defined by combinations of the deformations of individual elements, their complexity and completeness depending on the number of nodes and elements used to describe the structure.

Matrices of Mass and Stiffness properties, boundary conditions etc. are assembled and used, for example, to perform an eigenvalue analysis.

There are many commercial finite element computer packages on the market, and most large scientific computer installations have more than one implemented. It is therefore more sensible to use an existing package than to write one's own, unless one has special needs or wishes to develop a new element.

When using a commercial package, it is important to select the correct element and to make sure that the freedoms of that element are realistically restrained, either by prescribed restraints or by surrounding elements. Other refinements are package dependent and are discussed for the implementation appropriate to the present analysis in the next section.

4.2.2 Implementation

The package used for this analysis is the version F of A.S.A.S. (Atkins Research and Development Limited) on the IBM 360/195 computer at the Rutherford Laboratory. Access and data storage is facilitated by the S.R.C. network of linked Prime computers, one of which is situated in the University of Surrey.

The package is a general one covering linear static stress analysis, natural frequency analysis and heat conduction analysis. A certain amount of error checking of the data is incorporated and a number of program control options are available. Full details are in the User Manual (Atkins 1976).

Firstly the element must be chosen in order to generate a suitable mesh (e.g. triangular or quadrilateral elements). The element used is QUS4, a shell element which models in-plane membrane and out of plane bending behaviour. It is a quadrilateral and has 4 nodes (one each corner) with six degrees of freedom at each node (3 translations and 3 rotations). The limitations of this element are that it produces a faceted structure, i.e. the curvature of the bell is only modelled by the positions of nodes, and that the geometry must be such that the shortest side of an element is not less than a seventh of the largest side. The first limitation is minimised by using as many elements as possible to model the structure, and the second by careful selection of the positions of nodes.

A better choice of element would be the GCS8, another quadrilateral shell element with 4 mid-side nodes in addition to the 4 corner nodes. With this element the curvature of the structure is modelled through these mid-side nodes. The maximum radius of curvature and side-length ratio is limited such that many more elements and nodes are needed than can be handled by the mesh generation program (FEMGEN) and even if the mesh were produced, the subsequent analyses would use more computer time than can be justified by the budget and time available.

To model the rim, the TUBE element is used; a two noded element which is compatible with the QUS4 element, and is used such that the two types of element share the ring of nodes at the large end of the bell. The mesh generation package FEMGEN is used. This package interactively accepts co-ordinates of points and commands which build a mesh based on these points. Points are joined by lines or arcs of circle, these lines and arcs form surfaces which are divided into elements, these surfaces collectively form the structure. Graphics options are included in FEMGEN so that the geometry of the structure can be visually checked during construction.

For a single brass instrument bell, FEMGEN saves little of the work required to input individual co-ordinates by hand; it is useful for the graphics routines, its user interface of subroutines and that it stores the information defining each element by its surrounding node numbers. The user subroutines enable the user to interface to the data stored by FEMGEN. A program has been written such that once a basic bell structure exists in a FEMGEN data file, the user can input a new set of nodal co-ordinates to create different bells i.e. in this chapter all bells are modelled by 21 rings each of 16 nodes, each bell having different sets of co-ordinates for the nodes but the same element inter-connection data.

The structure must be restrained to prevent rigid body modes and to model struts or stays. Properties such as material constants and wall thicknesses must also be input.

A number of parameters can thus be investigated as to their effect on the natural mode shapes and frequencies of brass instrument bells:

(a) Material properties: Young's modulus, Poisson's ratio and density.

(b) Wall Thickness.

(c) Position of stay.

(d) Geometric shape.

(e) Rim size.

All these aspects are investigated in this chapter.

The data file for submission to A.S.A.S. is created on one of the Prime computers at the Rutherford Laboratory. It is submitted by surrounding it with appropriate Job Control Language and use of software available on the Prime for submission to the IBM machine. A typical job will take just under 4 minutes to run and use about 750K of core. Because of these requirements the job must be submitted to run during the night, thus mistakes and modifications are very costly in time. The output for one run is a very large text file (nearly 14,000 lines in length) which is routed back to the Prime from the IBM. This file can then be transferred through the network to the Prime computer at the University of Surrey, where all the unwanted lines can be deleted leaving pure data of the mode shapes and frequencies. These are in the form of a table; 8 modes across a page, listed first are the frequencies (one per mode) followed by 3 normalised displacements and 3 normalised rotations per node over the whole structure (336 nodes) to describe the mode shape. The 16 lowest frequency modes have been requested so there is a total of 32,272 numbers to be handled for just one run.

Once edited, the output data file continues its journey to the Nova 4 computer in the Physics Department, University of Surrey, for further analysis.

4.3 Data Preparation

4.3.1 Geometric Properties

The finite element mesh to describe the bell shapes consists of 336 nodes and 336 elements (320 QUS4, 16 TUBE). These numbers represent the best compromise between fineness of mesh, limitations of elements available, limitations to structure size in FEMGEN, handling the output data, computer time budget and time limitation of the project. The 336 nodes are allocated to 21 rings, each of 16 nodes; an example trombone mesh is shown in figure (4.1).

A number of methods of measuring brass instrument bell shapes are reviewed by Goodwin (1981). For the purposes of this chapter there is little incentive for accurate measurement of bell shape; large differences are being investigated, the difference between nominally identical instruments can be quite large and bells are often not circular in cross section. Two trombone shapes have been measured in the laboratory with dial calipers and an approximate wall thickness correction. The trombones used are a Boosey and Hawkes Imperial (serial no. 591802) and a Boosey and Hawkes Sovereign 937 (no serial no.), the Imperial bell being larger than the 937 at the bell throat and the bell mouth, the 937 bell being fatter in the middle.

Three sets of wall thickness measurements typical of such bells were supplied by Boosey and Hawkes Ltd. corresponding to bells made from 3 different thicknesses of brass sheet (known as 6 lb, 7 lb and 8 lb). This data is interpolated to provide a wall thickness profile along the length of the bell. The trombone data including stay position is presented in figure (4.2).

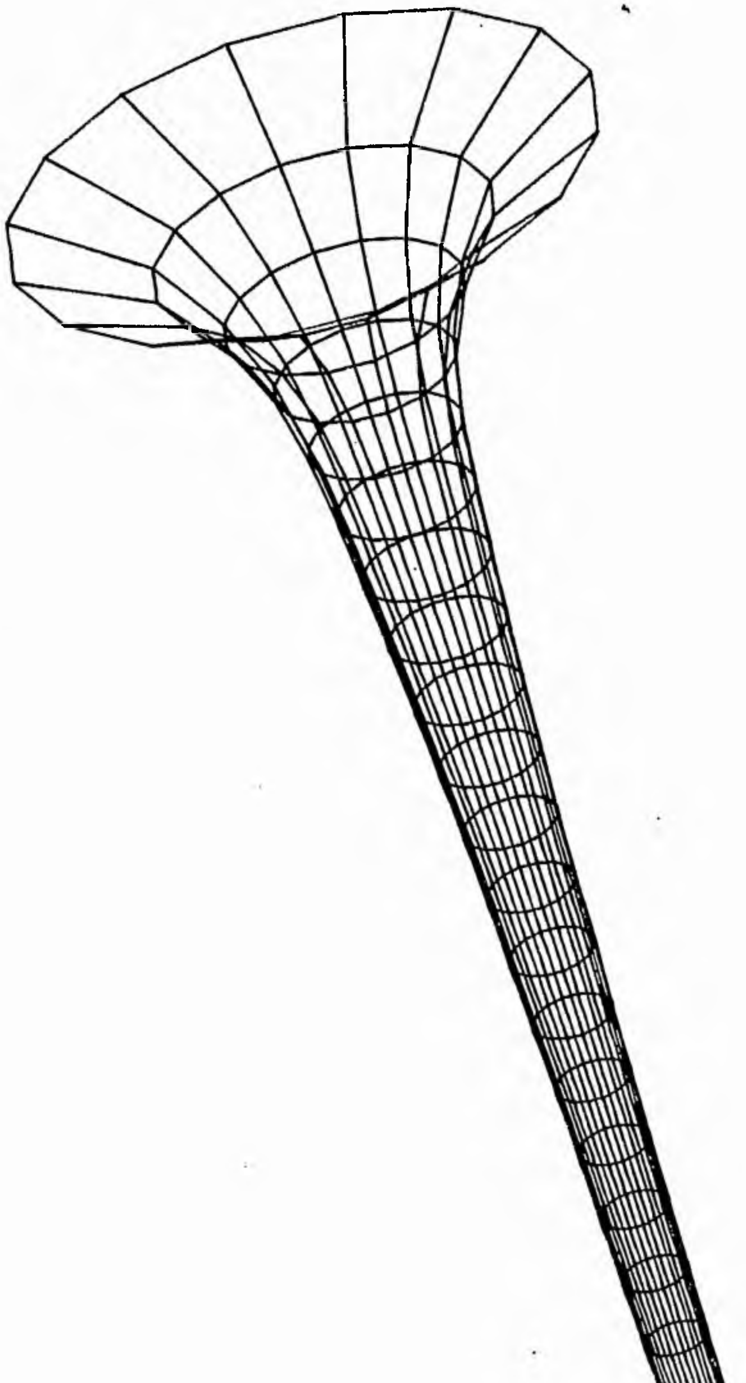


Figure 4.1. Trombone bell mesh.

Imperial Bell length	Imperial Bell radius	SOV Bell length	SOV Bell radius	Wall Thickness thin $\times 10^{-4}$ m	Wall Thickness medium $\times 10^{-4}$m	Wall Thickness thick $\times 10^{-4}$m
0	0.0105	0	0.01	3.81	4.57	5.59
0.0269	0.011	0.0258	0.0102	3.79	4.55	5.57
0.0538	0.0116	0.0516	0.0107	3.76	4.52	5.55
0.0807	0.0121	0.0774	0.0112	3.73	4.49	5.51
0.1076	0.0126	0.1032	0.0117	3.71	4.48	5.50
0.1345	0.0131	0.128	0.0123	3.69	4.47	5.48
0.1614	0.0137	0.1548	0.013	3.66	4.46	5.46
0.1883	0.0142	0.1806	0.0138	3.64	4.46	5.44
0.2152	0.0147	0.2064	0.0149	3.61	4.46	5.42
0.2421	0.0153	0.2322	0.0153	3.59	4.46	5.41
0.269	0.0158	0.2580	0.0163	3.56	4.46	5.39
0.2958	0.0168	0.2838	0.0176	3.54	4.46	5.37
0.3227	0.0179	0.3096	0.0188	3.51	4.46	5.35
0.3496	0.0195	0.3354	0.0204	3.49	4.46	5.33
0.3765	0.0211	0.3612	0.022	3.46	4.46	5.31
0.4034	0.0239	0.3870	0.0239	3.44	4.46	5.29
0.4303	0.0277	0.4128	0.0264	3.41	4.46	5.27
0.4572	0.0326	0.4386	0.031	3.09	4.32	4.61
0.484	0.0415	0.4644	0.0411	3.10	4.33	5.59
0.511	0.0616	0.4902	0.0582	3.56	4.57	5.59
0.5379	0.1032	0.516	0.1013	3.56	4.57	5.59

Stay

Figure 4.2.

Trombone bells geometric data

Boosey and Hawkes Ltd. also supplied the shape of a Sovereign trumpet bell and two sets of wall thickness measurements corresponding to two different methods of manufacture. These measurements are interpolated to generate data for the finite element mesh and are presented in figure (4.3). The mesh for the trumpet bell is shown in figure (4.4).

Rim sizes and shapes vary a lot between instruments, depending on method of formation and dimensions of materials used. All rims are formed by folding back the bell end, and usually brass or other wire is inserted in this "hem". If no wire is inserted the rim tends to be flat; with wire, cylindrical. Many rims have been measured using dial calipers and typical average values calculated. The TUBE element requires an outside diameter and a wall thickness as geometric properties, so a wall thickness very nearly half the diameter is used.

Complete tables of the data used for each case analysed are presented in figure (4.5, a and b).

For all cases, the small end of the bell is considered to be held rigid.* This is a sufficient boundary condition to suppress all rigid body modes. For a trombone this is a good approximation as the small end of the bell is soldered into a thick metal band held by a stay. For a trumpet bell the approximation is not so good as the bell is continuous back from the portion analysed, through a 180° bend and into the valve casing. However for many of the modes most of the vibration energy is in the large end of the bell so the error of the approximation should not be too great.

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* This implies that deflection and slope are zero, and in the author's opinion this is a good approximation.

Bell Length m	Bell radius m	Wall thickness thin $\times 10^{-4}$m	Wall thickness thick $\times 10^{-4}$m
0	0.011	3.56	3.81
0.0127	0.0113	3.56	3.81
0.0254	0.0116	3.56	3.81
0.0381	0.0119	3.43	3.81
0.0508	0.0121	3.30	3.81
0.0635	0.0124	3.43	3.81
0.0762	0.0127	3.56	3.81
0.0889	0.013	3.68	3.81
0.1016	0.0133	3.81	3.81
0.1143	0.014	3.56	3.68
0.127	0.0148	3.30	3.56
0.1397	0.0159	3.17	3.30
0.1524	0.017	3.05	3.05
0.1651	0.0185	2.79	3.05
0.1778	0.0206	2.79	3.81
0.1905	0.0242	3.56	3.81
0.2032	0.0298	3.56	3.05
0.2095	0.0359	3.68	3.05
0.2159	0.042	3.81	3.05
0.2222	0.0519	3.68	3.81
0.2286	0.0619	3.56	4.57

Stay

Figure 4.3.

Trumpet bells geometric data

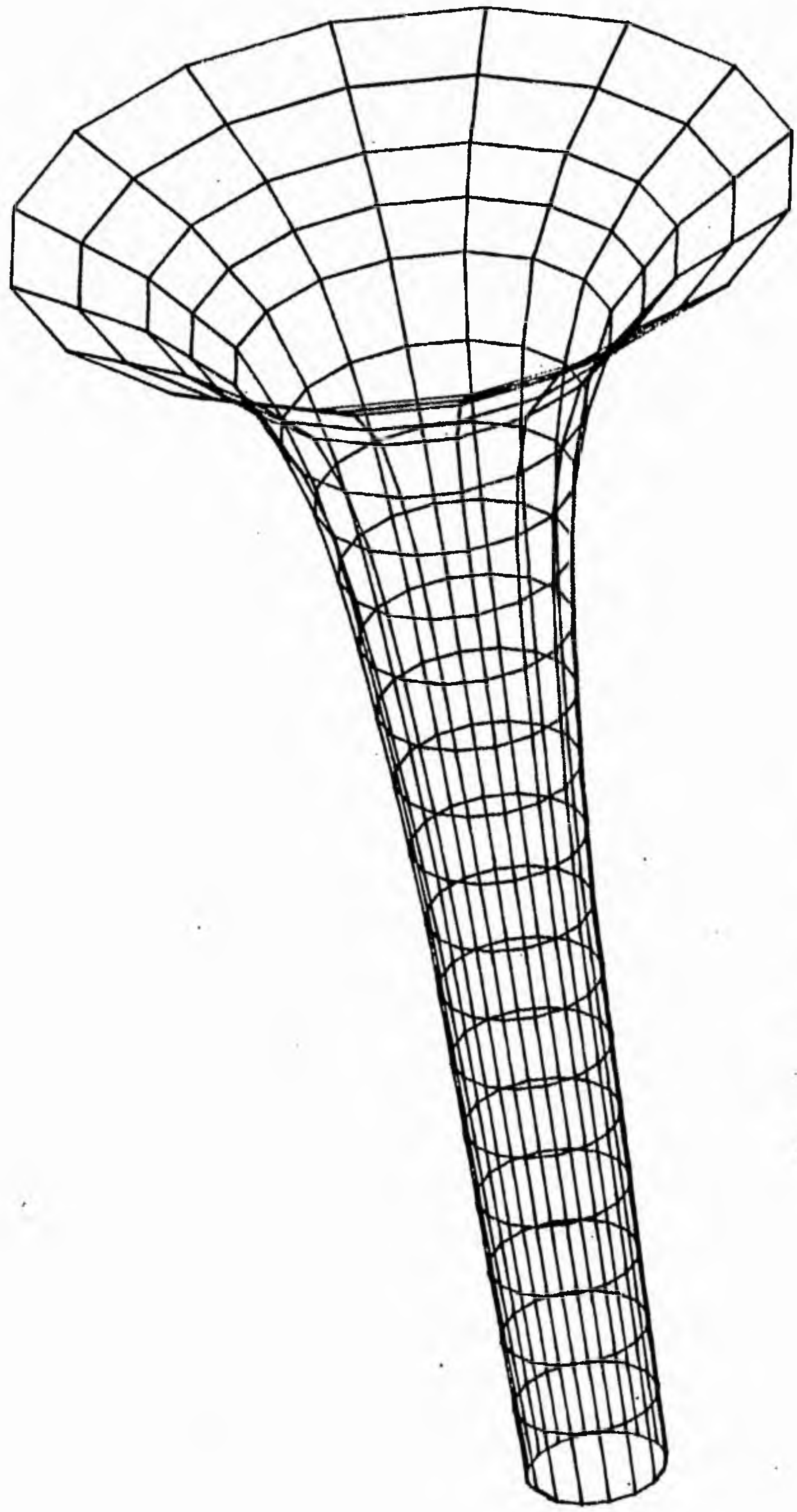

Figure 4.4 Trumpet bell mesh

Code	Young's Modulus NM^{-2}	Poisson's Ratio	Density $Kg.m^{-3}$	Wall Thickness Type	Rim Outside Diameter mm	Bell shape	Feature
SOVC3	1.12E11	0.35	8500	Thin	2.8	Sovereign 937	Thin trombone bell
SOVC4	1.12E11	0.35	8500	Medium	2.9	Sovereign 937	Medium trombone bell
SOVC5	1.12E11	0.35	8500	Thick	3.2	Sovereign 937	Thick trombone bell
SOVD4	7.0 E10	0.37	10500	Medium	2.9	Sovereign 937	Silver trombone bell
SOVE4	1.17E11	0.35	8900	Medium	2.9	Sovereign 937	Copper trombone bell
SOVF4	1.3 E11	0.33	8700	Medium	2.9	Sovereign 937	Nickel Silver Trombone bell

Figure 4.5 a Finite Element Data Cases

Code	Young's Modulus NM^{-2}	Poisson's Ratio	Density Kg m^{-3}	Wall Thickness Type	Rim Outside Diameter mm	Bell shape	Feature
SOVG4	1.12E11	0.35	8500	Medium	2.9	Sovereign 937	Forward stay
SOVH4	1.12E11	0.35	8500	Medium	4.0	Sovereign 937	Thick rim
SOVI4	1.12E11	0.35	8500	Medium	-	Sovereign 937	No rim
IMPC4	1.12E11	0.35	8500	Medium	2.9	Imperial	Different shape bell
SOVR4	1.12E11	0.35	8500	Thin	2.0	Sovereign trumpet	Thin trumpet bell
SOVR5	1.12E11	0.35	8500	Thick	2.0	Sovereign trumpet	Thick trumpet bell

Figure 4.5 b Finite Element Data Cases

The stay on the trombone is modelled by holding a total of 10 nodes rigid (five on each of two consecutive rings of nodes), for the trumpet bell 14 nodes (two on each of seven consecutive rings of nodes) are held rigid.

4.3.2 Material Properties

A table of material properties can be found in appendix C. Typical values are chosen for brass, silver, copper and nickel silver to investigate the effect of different materials, where a range is given an average value is used. Despite various degrees of work hardening in a bell, work hardening affects properties governing permanent deformation, not the elastic properties (Chalmers 1951). The material properties used are displayed in figure (4.5 a and b).

4.3.3 Summary

The finite element method has been chosen as the best method on which to base a modal analysis of brass instrument bells. This is because a brass instrument bell is of a mathematically arbitrary shape and therefore does not lend itself to analytic techniques, and that the finite element method is relatively simple due to the many computer packages available. A suitable package has been chosen and the required data collated to form a set of cases designed to investigate the effect of bell parameters. The frequencies and shapes of the lowest frequency modes for each set of data have been extracted and are presented in section 4.4. Section 4.5 carries the analysis further to attempt to predict the relative levels of excitation of these modes.

4.4 Mode Frequencies and Shapes

4.4.1 Trombone Bells

Exploiting the user subroutines of FEMGEN, the displacements describing the mode shapes can be visually displayed. A program has been written which will read in the nodal co-ordinates from the FEMGEN database, read the displacements pertaining to a specified mode from the edited output file, shift the nodal co-ordinates by these displacements suitably scaled and write the new set of co-ordinates away in the FEMGEN database under a new structure name. The FEMGEN graphics routines can then be used to display the distorted bell. A set of 16 mode shapes are drawn in figure (4.6 a, b, c and d).

This set of mode shapes is for the case SOVC4 (as in figure 4.5 a) in ascending frequency order, but is typical of all the trombone data sets described in section 4.3, though for the cases where the structural geometry is very different (e.g. SOVI4; with no rim) the mode shapes are in a different ascending frequency order. The frequencies of the natural modes for the cases analysed are presented in figure (4.7).

To view the modes first decide whether the bell is facing towards you or away from you; trying to view it both ways will help understanding of the mode shape. Starting at the small end look at the shape of each individual ring of nodes along the bell, for example note the ovality of rings 10 and 11 (1 being the ring at the small end) being different than that expected from just the effects of perspective. Also take careful note of the shape of rings 17 and 18 as they can be swamped by the grossly distorted shape of ring 21 (the large end of the bell), particularly in modes such as G or H.

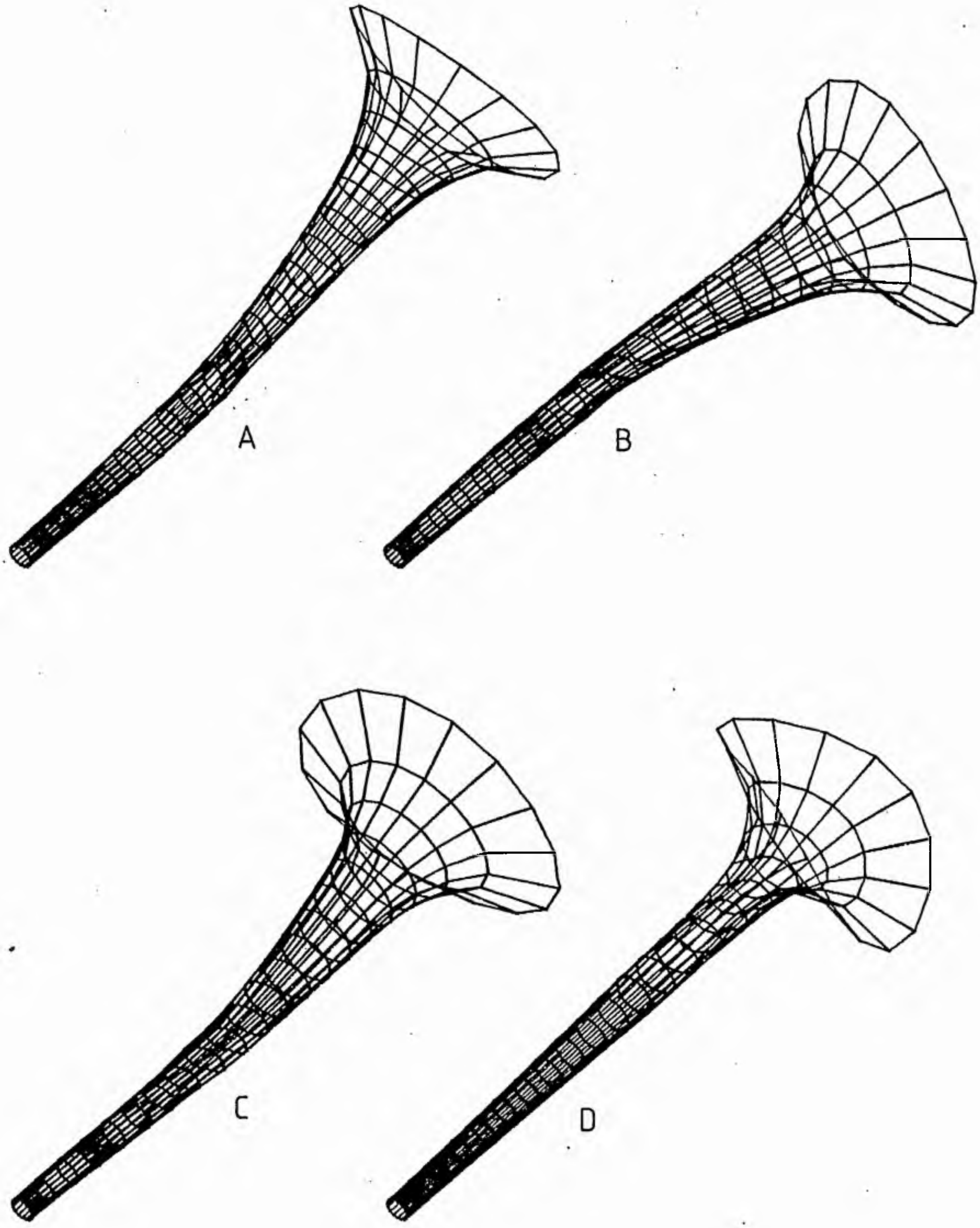

Figure 4.6 a Trombone Bell Mode Shapes.

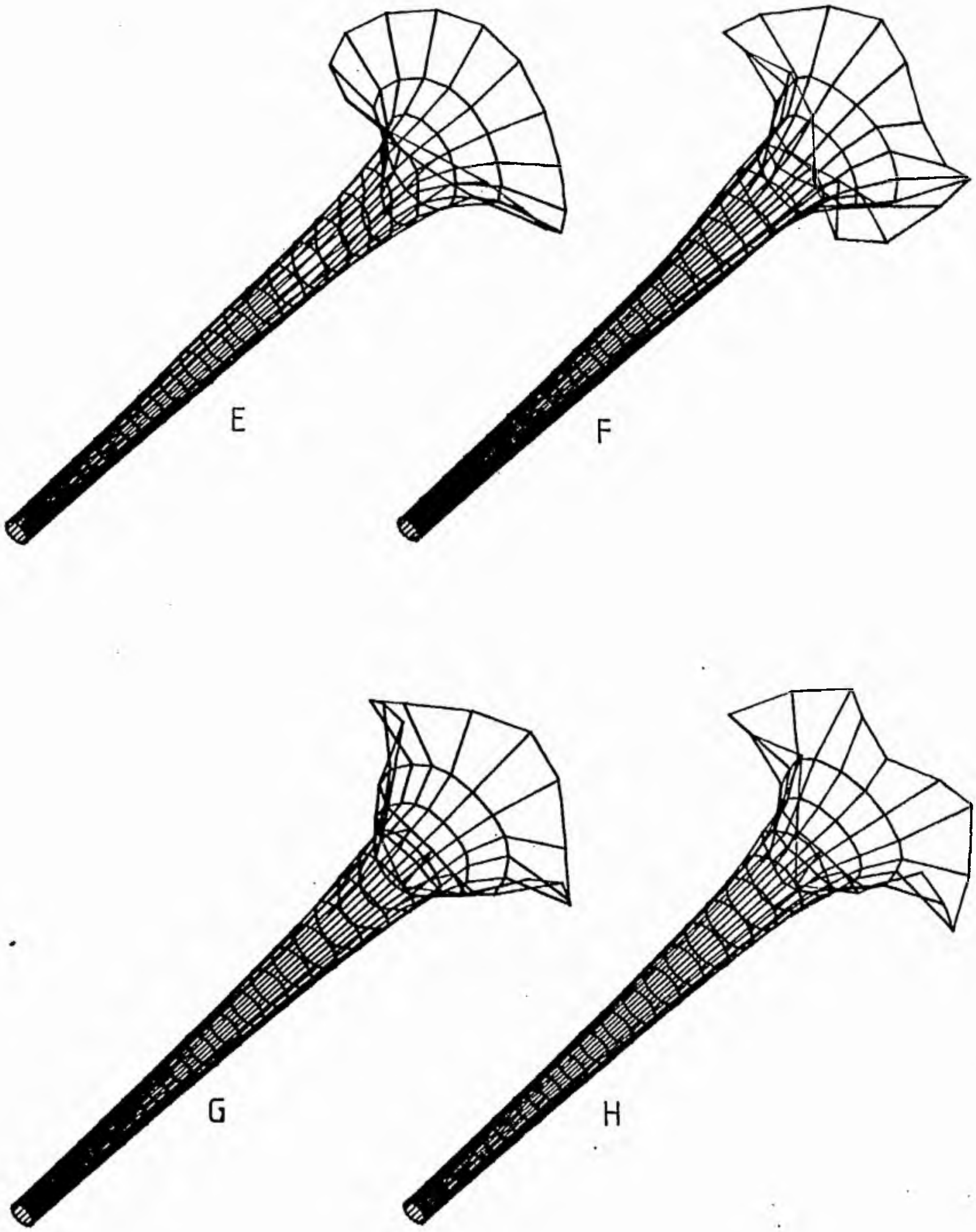

Figure 4.6 b Trombone Bell Mode Shapes.

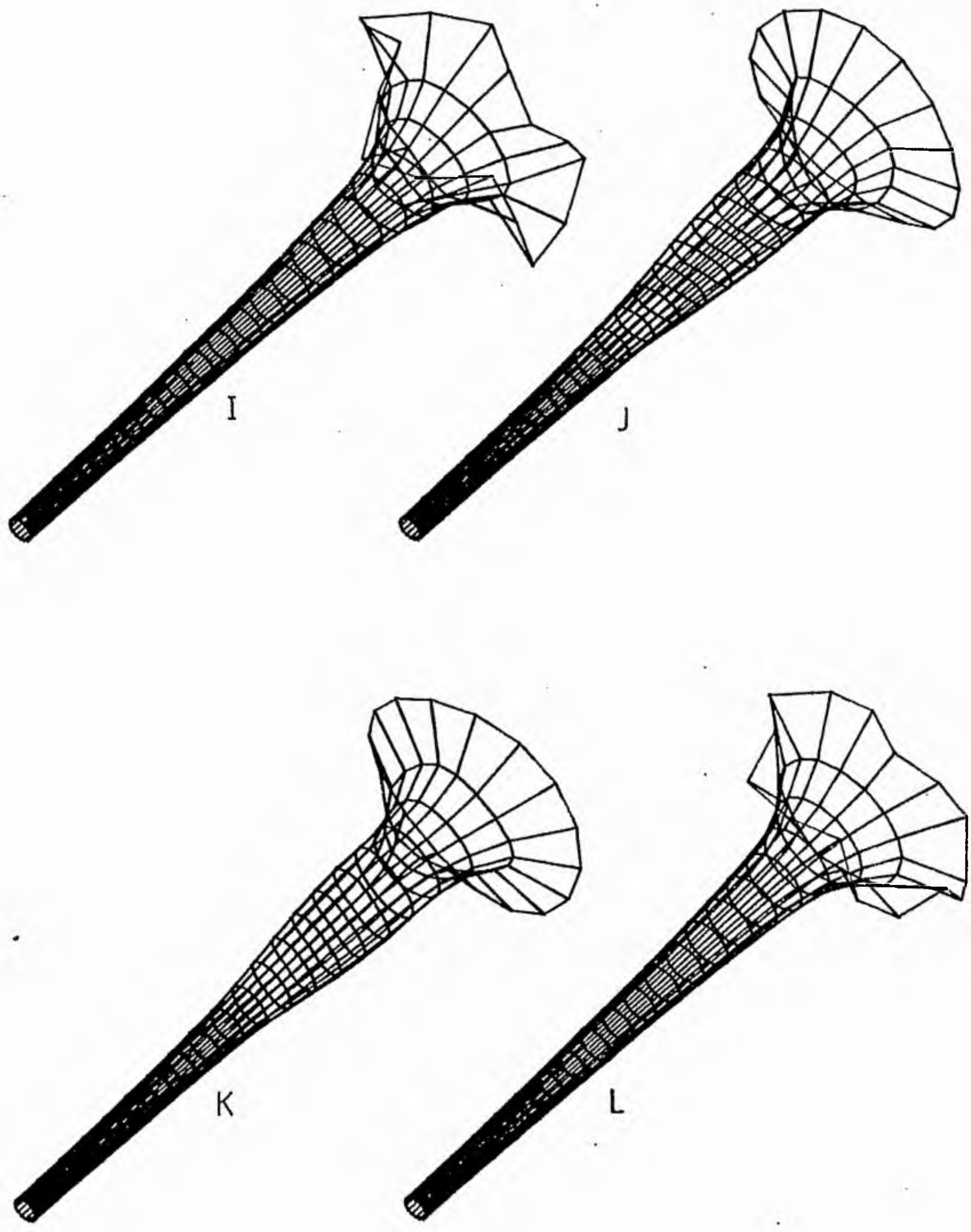

Figure 4.6 c Trombone Bell Mode Shapes.

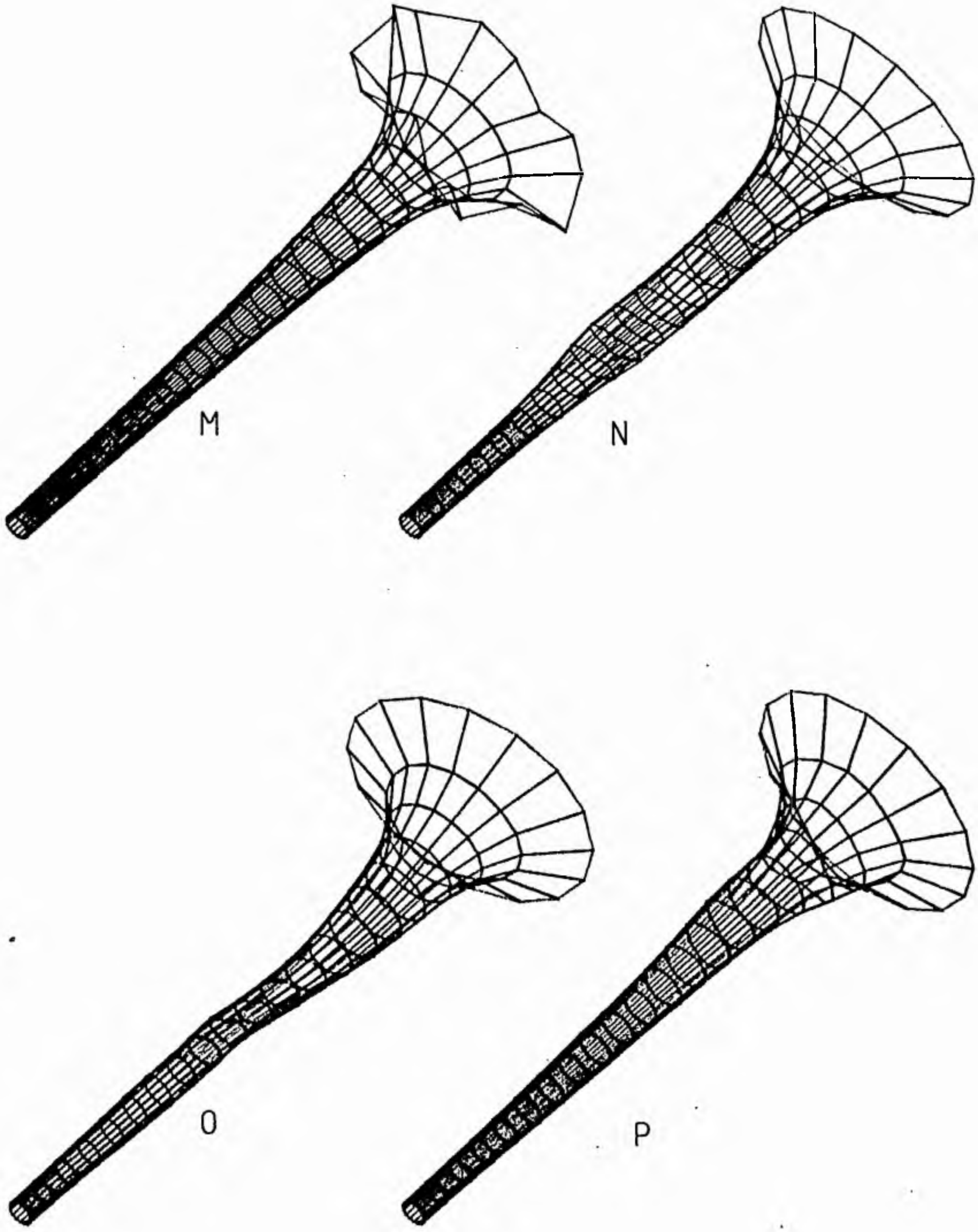

Figure 4.6 d Trombone Bell Mode Shapes.

Mode Number	SOVC3	SOVC4	SOVC5	SOVD4	SOVE4	SOVF4	SOVG4	SOVH4	SOVI4	IMPC4
1	86.32	88.26	89.94	62.84	88.16	93.91	11.4	81.3	98.6	81.73
2	93.09	95.04	96.84	67.53	94.93	108.3	125.1	87.3	106.7	88.43
3	141.5	145.7	149.1	103.3	145.5	155.7	181.6	130.1	157.5	139.1
4	165.6	194.0	207.6	138.4	193.7	205.3	197.4	192.1	158.2	167.1
5	166.2	194.8	208.5	139.0	194.5	206.2	202.0	193.0	172.2	168.7
6	239.0	256.5	282.8	182.1	256.2	273.0	256.8	345.2	173.5	347.8
7	239.2	256.7	282.9	282.2	256.3	273.1	257.0	345.4	173.7	347.9
8	381.0	385.7	427.8	274.1	385.2	410.8	385.7	493.7	217.9	396.5
9	381.4	386.0	428.2	274.3	385.5	411.1	386.0	500.0	218.4	397.3
10	428.8	472.0	509.3	336.2	471.2	500.6	518.1	598.3	245.2	492.3
11	437.6	478.6	514.7	348.8	477.8	507.6	551.9	599.0	245.8	495.2
12	603.1	609.4	678.1	433.3	608.7	649.1	609.5	701.7	432.4	585.7
13	603.2	609.6	678.4	433.3	608.9	649.2	609.7	772.8	433.2	586.2
14	690.5	740.2	767.8	526.9	739.1	785.9	829.8	911.4	458.2	606.9
15	788.1	822.5	825.2	583.6	821.2	875.6	874.3	912.9	465.3	608.5
16	788.6	873.2	915.7	620.4	871.3	924.1	875.0	947.1	714.2	718.3

Frequencies in Hz

Figure 4.7 Trombone Bell Mode Frequencies.

The modes can generally be characterised by the number of "cycles" of displacement both axially and circumferentially, though as the displacements at the small end are too small to be seen there is uncertainty in the axial direction. However, the circumferential patterns are abundantly clear, for example mode F is characterised by 3 cycles, mode H by 4.

Many of the mode shapes and frequencies come in pairs. This is due to the asymmetry provided by the stay, which implies that the frequency of a particular mode shape will depend upon its geometrical relation to that stay, for example a mode shape such as A vibrating in the plane of the stay or normal to the stay. The two extremes of frequency for this mode shape are described by these two directions, one being rotated by 90° about the bell axis to the other, any other direction of vibration can be described by a vector sum of these two and will have an intermediate frequency.

It can be seen in figure (4.7) that the frequencies of these modes lie in the full musical range of frequencies used by the trombone, so there is an initial opportunity for these modes to contribute to the musical quality of the instrument.

4.4.2 Trumpet Bell

Using the same technique as for the trombone bells, the mode shapes for the trumpet bells can be visualised and are presented in figure (4.8 a, b, c and d). The set of mode shapes is typical for both the thick and thin bells. A table of frequencies for these modes is presented in figure (4.9).

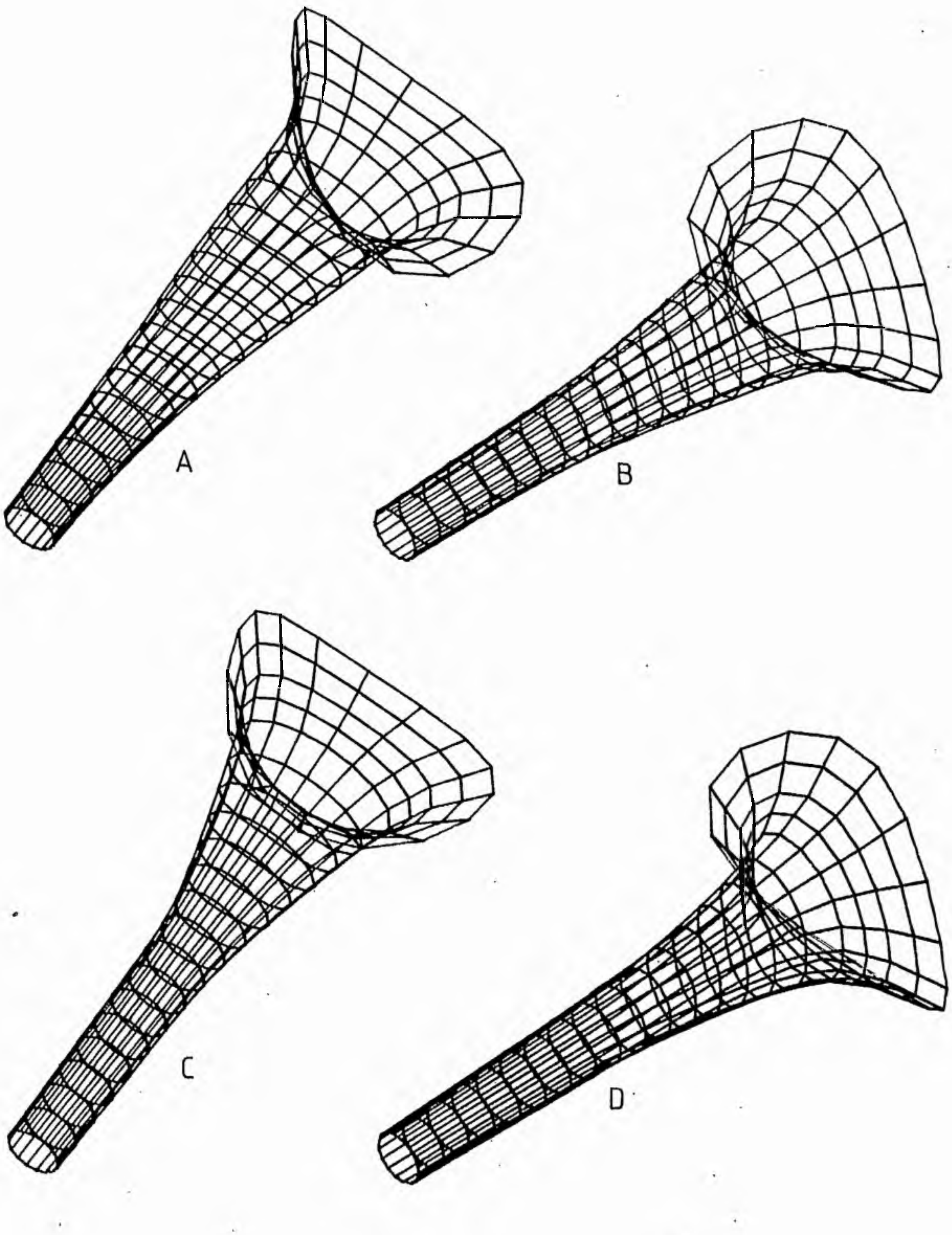

Figure 4.8 a Trumpet Bell Mode Shapes.

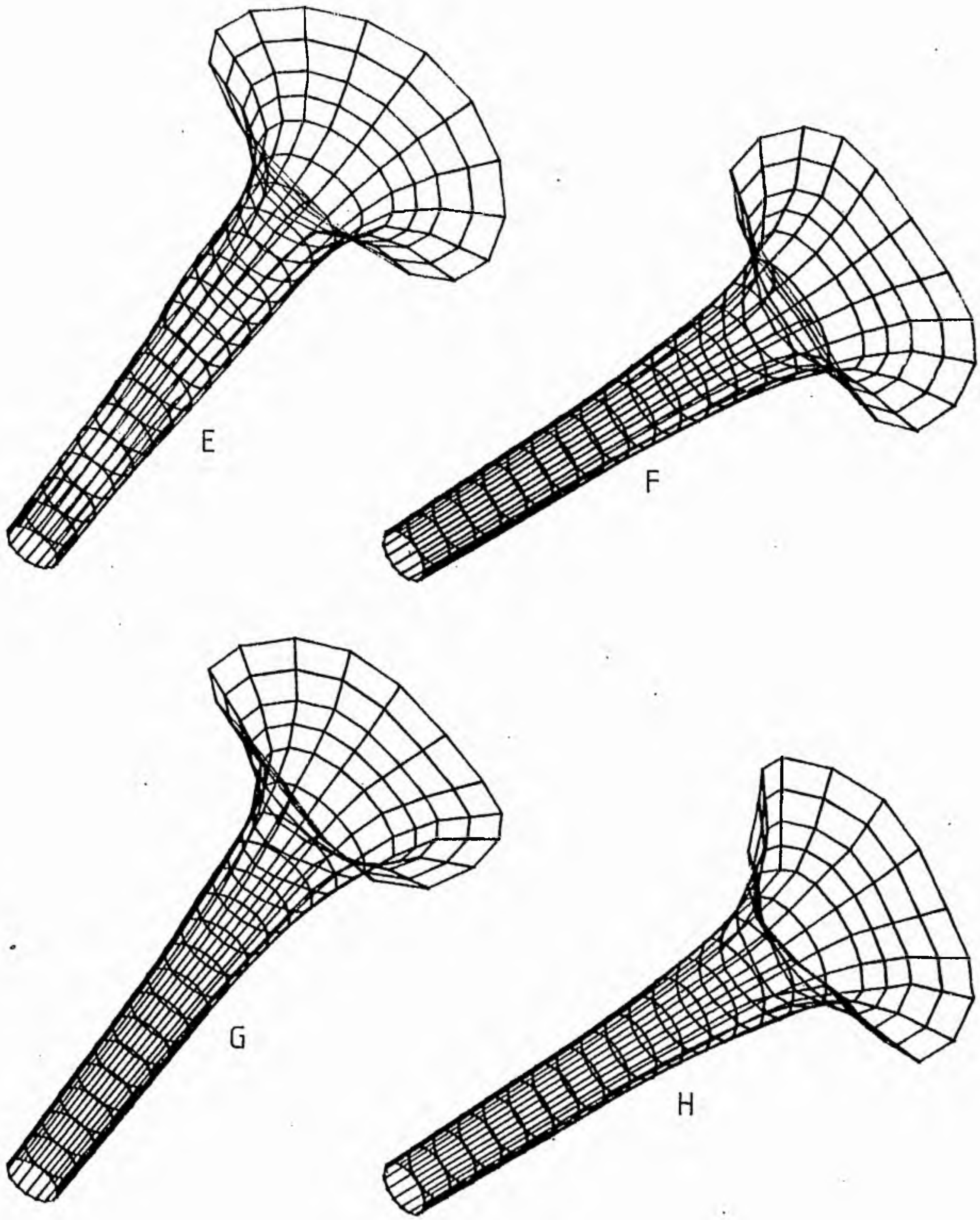

Figure 4.8 b Trumpet Bell Mode Shapes.

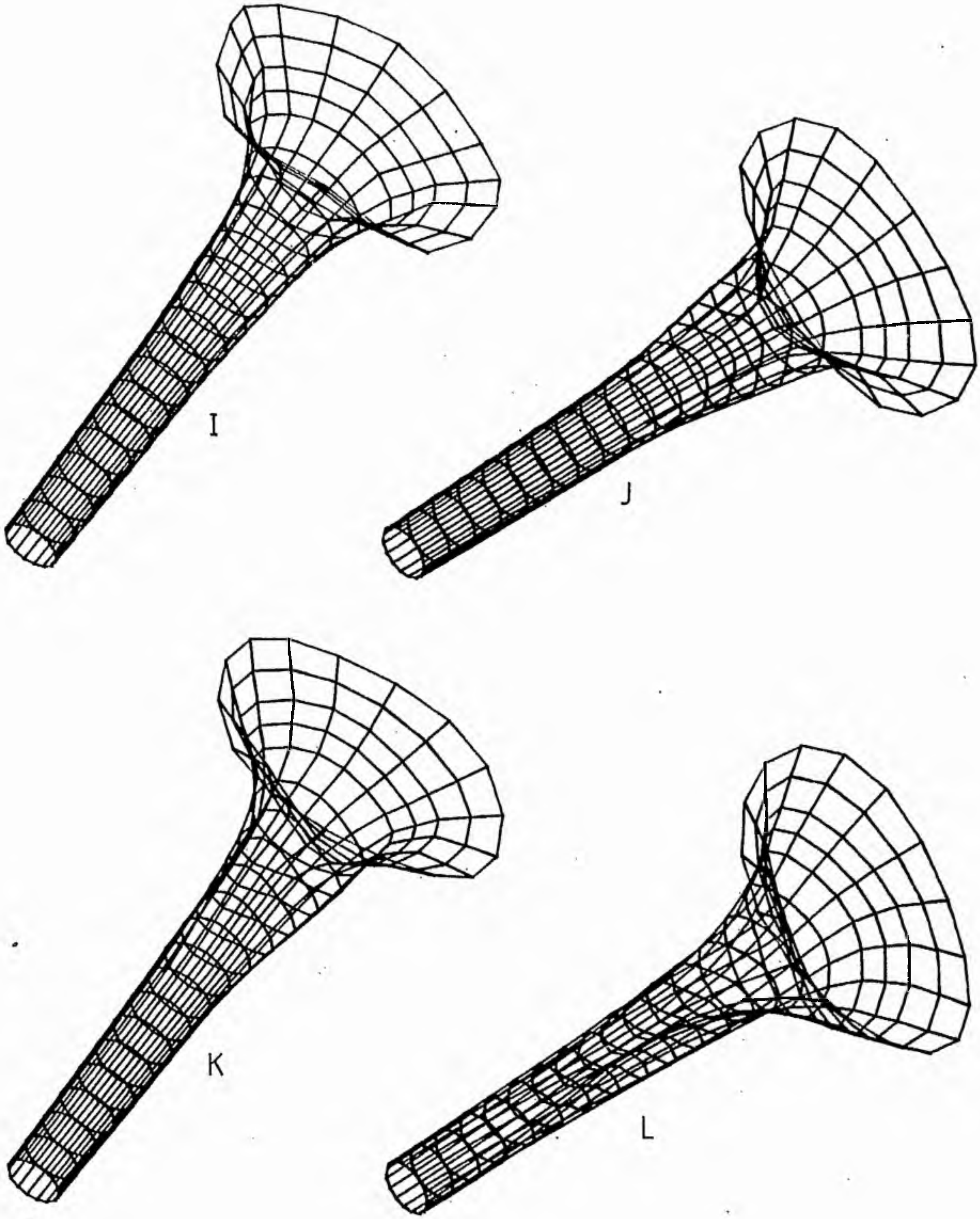

Figure 4.8 c Trumpet Bell Mode Shapes.

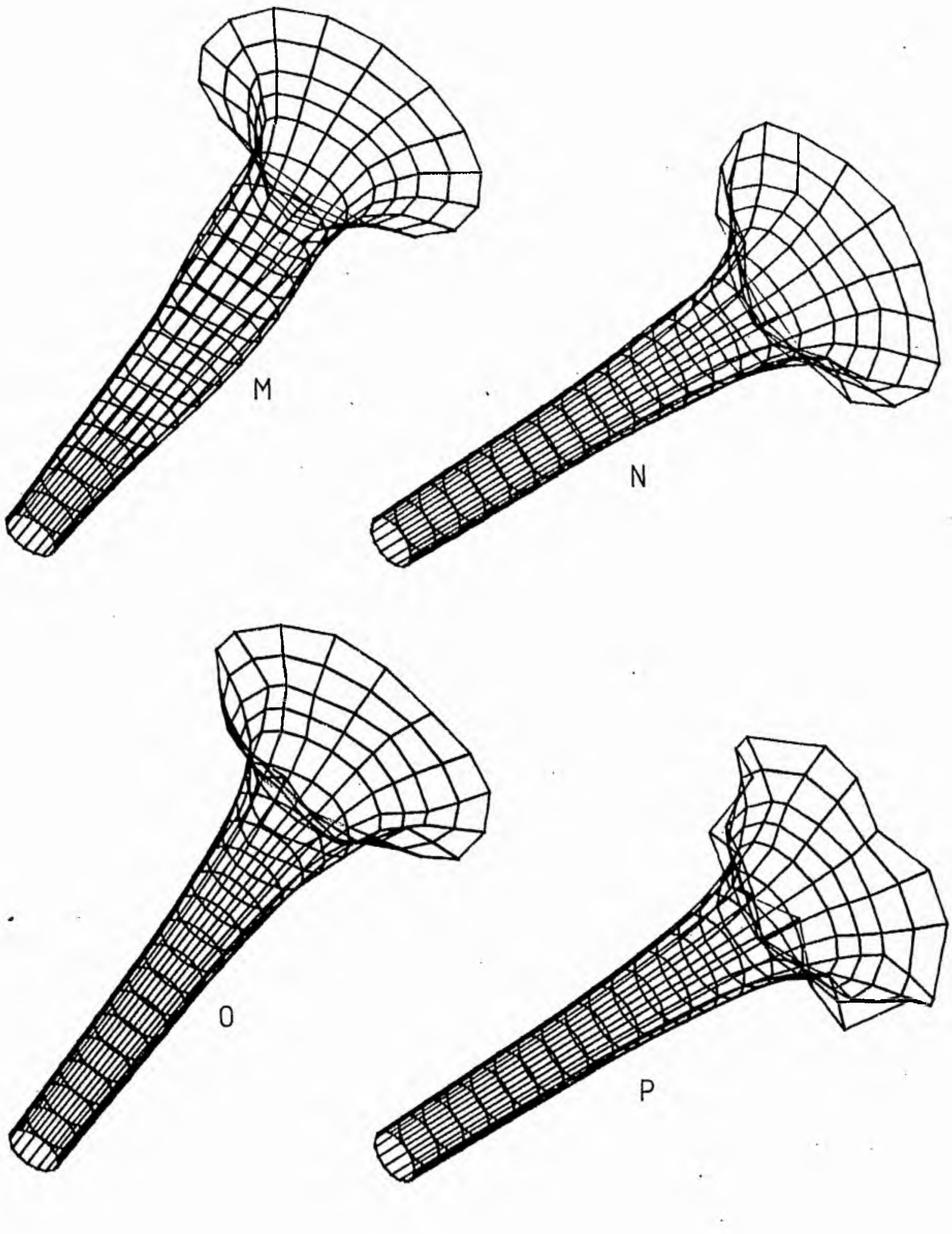

Figure 4.8 d Trumpet Bell Mode Shapes.

Mode No	SOVR4	SOVR5
1	247.4	257.7
2	287.7	297.4
3	389.6	343.9
4	389.9	394.1
5	530.1	541.8
6	621.5	617.5
7	621.7	617.7
8	1055.7	1011.9
9	1056.8	1014.2
10	1109.7	1196.1
11	1110.4	1198.8
12	1143.6	1206.4
13	1162.2	1209.4
14	1356.6	1338.7
15	1358.0	1344.4
16	1561.4	1465.1

Frequencies (Hz)

Figure 4.9 Trumpet Bell Mode Frequencies.

All of the comments on the trombone bell mode shapes apply to the pictures in figure (4.8), though these modes are not so clear as to their geometry. Similarly the frequencies of these modes lie in the musical playing range of the instrument.

The condition, discussed in section 4.3.1, that the small end of the bell is considered to be held rigid will have little effect on the natural mode shapes of a trumpet bell, but the frequencies will be a little high, experience indicates perhaps by as much as 10%.

In the next section calculations will be made to deduce how much these modes are excited during acoustic excitation.

4.5 Response of Brass Instrument Bells

4.5.1 Introduction

In order to calculate the response of brass instrument bells, a generalised method of vibration analysis is used. The principles are discussed in detail in Skudrzyk (1968), and the method and notation used below is as in Clarkson (1977).

The method is based on two equations:-

$$W(x,y,z,t) = \sum_{r=1}^{\infty} q_r(t).f_r(x,y,z) \qquad (4.1)$$

Where W represents the deflected shape of the structure, q_r the amplitude and time dependence of a particular mode of vibration and is known as a Generalised Co-ordinate, and $f_r(x,y,z)$ is the normalised mode shape of the r th natural mode of vibration.

Equation (4.1) implies that any deflected shape of the structure can be represented as a sum of its natural modes of vibration.

The second is:

$$M_r\ddot{q}_r + C_r\dot{q}_r + K_rq_r = L_r(t) \tag{4.2}$$

Equation (4.2) is of similar form to that of a mass-spring-damper system except in the generalised case M_r is the Generalised Mass, C_r the Generalised Damping Coefficient, K_r the Generalised Stiffness and L_r the Generalised Force.

This method of analysis is suitable for interfacing to the output from the finite element package as the mode shapes that are output are simply the $f_r(x,y,z)$'s as in equation (4.1). Equation (4.2) can be solved by the standard solution:

$$W(x,y,z,t) = \sum_{r=1}^{\infty} |\alpha_r| L_r f_r(x,y,z)\cos(\omega t - \phi_r) \tag{4.3}$$

where

$$\alpha_r = \frac{1}{(K_r - M_r\omega^2) + j\omega C_r} \tag{4.4}$$

ω = angular frequency of excitation.

Although the finite element package does not output an infinite number of f_r's, a good approximation can be obtained with the 16 modes output, up to the frequency of the highest frequency mode.

The results from this analysis are for two purposes:

(a) To compare with the experimental results of Kitchin (1980). Bells similar to the thick, medium and thin Sovereign bells analysed by the finite element package were acoustically excited and their mechanical response measured at a fixed point using a Laser-Doppler Velocimeter. The bells were not on complete instruments, but were fixed at the small end and supported by a cotton thread sling approximately 11 cm from the large end. Three finite element analyses were performed similar to SOVC3, SOVC4 and SOVC5 except the stay was not modelled and the sling support was modelled with TUBE elements of small circumference. It is not possible with A.S.A.S. to model the static forces imposed by this support. The experimental results show a strongly excited mode around 250 Hz for the medium bell (206 Hz for the thin bell, 270 Hz for the thick bell), and a weaker less consistent mode at higher frequency (500-700 Hz).

The measurements were made using the Nova 4 computer and acoustic driver facilities at the Physics Department, University of Surrey, so the data and equipment (except the Laser-Doppler Velocimeter) are readily available.

(b) To compare the results generated by varying the geometric and material properties of the bells.

The next two sections discuss the implementation of the method described in this section.

4.5.2 Calculation of Parameters

The Generalised Mass is defined to have the same magnitude of that mass which when moving with velocity $\dot{q}_r$ has the same kinetic energy as the whole system when moving with velocity $\dot{q}_r f_r(x,y,z)$.

For a brass instrument bell:

$$M_r = \int_A \mu(f_r^2(x) + f_r^2(y) + f_r^2(z)).dA \qquad (4.5)$$

where A is the surface area and μ the mass per unit area.

This parameter has a unique and constant value for each natural mode of vibration.

A further contribution to the kinetic energy will come from the rotations. An output from A.S.A.S. of all the lumped masses and inertias associated with each node for a trombone bell was obtained. This showed that the inertias are several orders of magnitude smaller than the masses, therefore for this simple analysis they will be ignored though it is acknowledged that any future extensions of the work described in this chapter should include their contribution.

A computer subroutine (MASS) evaluates equation (4.5) by a discrete approximation:

$$M_r = \sum_{n=1}^{NN} \rho.A_n.T_n(f_r^2(x) + f_r^2(y) + f_r^2(z))_n \qquad (4.6)$$

where ρ is the density of the material, A_n the area for which the mass per unit area is to be lumped at node n, T_n the average thickness over that area and NN is the number of nodes describing the structure. A subroutine AREA must be called before MASS to calculate the effective areas for each node. The contribution to M_r made by the rim is calculated on a similar principle and added separately.

The Generalised Stiffness is defined as the stiffness of the linear spring which when displaced by q_r from its unstrained position, has the same potential energy stored within it as the actual system when displaced by $q_r f_r(x,y,z)$. However, as the frequency of each mode is known, the generalised stiffness is implied from the equation:

$$K_r = \omega_r^2 M_r \qquad (4.7)$$

This is implemented in subroutine FSTIFF.

The Generalised Damping Coefficient is defined as the rate of that damper which, when extended at velocity $\dot{q}_r$, dissipates energy at the same rate as the whole system of damping forces and pressures on and within the system, when moving with velocity $\dot{q}_r f_r(x,y,z)$.

This is the most difficult of all the parameters to evaluate. Very little data is available for values of internal damping of the material. Although in the analysis parts of the bell are held rigid, it is not known how much energy is lost down the stay, for example on a trombone where there is usually a "lossy" hand on the other end of the bell stay. Radiation damping calculations will be very complex due to the acoustically complex shape of the radiating surface (viz. the bell). For the purposes of this simple analysis the values of damping calculated from experimental results by Kitchin (1980) are used. His values are all of a similar order of magnitude irrespective of frequency such that a straight line fit to points on a graph of damping versus frequency is justified. Values of damping are generated in this way by subroutine DAMP using the equation:

$$C = \frac{{\omega_r}^2}{\omega} M_r \eta \qquad (4.8)$$

where ω is the frequency of the forcing pressure and η is the damping factor based on the measurements of Kitchin (1980). Equation (4.8) is based on the principles as presented by Skudrzyk (1968).

These parameters are sufficient for the solution of equation (4.4). The final parameter to be evaluated is the generalised force and is discussed in the next section.

4.5.3 The Forcing Function

The Generalised Force is defined as that single force which when moved through the "virtual" displacement δq_r, does the same amount of work as all the external applied forces and pressures acting on the system when the system is moved through the virtual displacement $\delta q_r f_r(x,y,z)$.

This leads to the equation:

$$L_r(t) = \int_A p(x,y,z,t) f_r(x,y,z).dA \qquad (4.9)$$

where $p(x,y,z,t)$ is the instantaneous pressure at the point x,y,z and at time t. The time dependence will now be assumed to be sinusoidal and $p(x,y,z)$ will be the amplitude of the oscillating pressure.

For the case of a brass instrument bell, the pressure amplitude will be assumed to be circularly symmetric, i.e. independent of x and z, but will vary along the axis of the bell (the y direction). The function $p(y)$ will vary with frequency but must be known in order to evaluate equation (4.9).

Measurements were made in a bell excited acoustically by the system described in section 3.3.1. Sound pressure was measured with a microphone (B and K type 4002) at 21 positions along the bell internal wall surface corresponding to the 21 rings of nodes in the finite element model, these pressures being normalised to a maximum value of 1.0. These measurements were repeated over a frequency range of 51 Hz to 1050 Hz at 1 Hz intervals, and thus a computer file of normalised pressure distributions over a suitable range was created. A distribution for a particular frequency is accessed from this large data file by the subroutine GETPROF. The collection and storage of this vast amount of data was facilitated by the program PROFRUN which supervised the control and measurements similar to the program PZTRUNO as in section 3.3.1.

The integral in equation (4.9) can now be evaluated by summing the contributions of force at each node:

$$L_r = P_0 \sum_{n=1}^{NN} p_n.f_n.A_n \tag{4.10}$$

where P_0 is the pressure amplitude, p_n is the value of the pressure distribution at node n, f_n is the normalised displacement normal to the bell surface at node n and A_n is the effective bell area for which the pressure acts on node n.

The summation part of equation (4.10) is a measure of the coupling of pressure amplitude P_0 to a particular mode shape and implies that for a "perfectly symmetric" mode shape (viz. for every positive component of displacement there is an equal negative component of displacement) the coupling will be zero and thus the mode will not be excited. If the geometry of the bell is perfectly symmetric about its axis, then the mode shapes output from the finite element package will be perfectly symmetric and thus it is predicted that the bell will not be excited into vibration at all. The coupling factor for a particular mode could be described as a combination of the nature of the excitation and a measure of the non-perfectness of that mode shape.

Although it is assumed that the excitation pressure is symmetric about the bell axis, if it is not this property will also increase the coupling between the excitation pressures and the displacements of the mode shapes.

For the results presented in section 4.5.5 a value of 1.0 is used for P_0 and thus the graphs in that section are for comparison purposes only.

For the results presented in section 4.5.4 (i.e. comparing with the experiments of Kitchin (1980)) a more complex pressure is used. Firstly the pressure distributions are re-normalised to a value of 1.0 at the throat of the bell rather than a maximum value of 1.0. Secondly, a file of calibrated pressure amplitudes measured in the throat of a bell excited in the same way as the bells used in the measurements of Kitchin (1980) was used to calculate a realistic excitation level. These two effects are combined to produce a computer file of values of P_0 for frequencies in the range 51 Hz to 1024 Hz, to be used in equation (4.10). A plot (on a logarithmic scale) of this pressure amplitude file is shown in figure (4.10).

The next two sections present the results for the methods described above applied to the various cases analysed by the finite element method.

4.5.4 Comparison with Experimental Results

To compare with the experimental results of Kitchin (1980), three cases were analysed by the finite element technique which as near as possible corresponded to the bells used in his experiments. He used bells made from brass sheet of three different thicknesses. The bells were supported by a cotton sling positioned about 11 cm from the large end of the bell, and were supported and driven acoustically at the bell throat. These bells correspond to those of data cases SOVC3, SOVC4 and SOVC5 with the modifications of no stay being modelled and the cotton sling modelled by TUBE elements of small dimensions (0.25 mm in diameter).

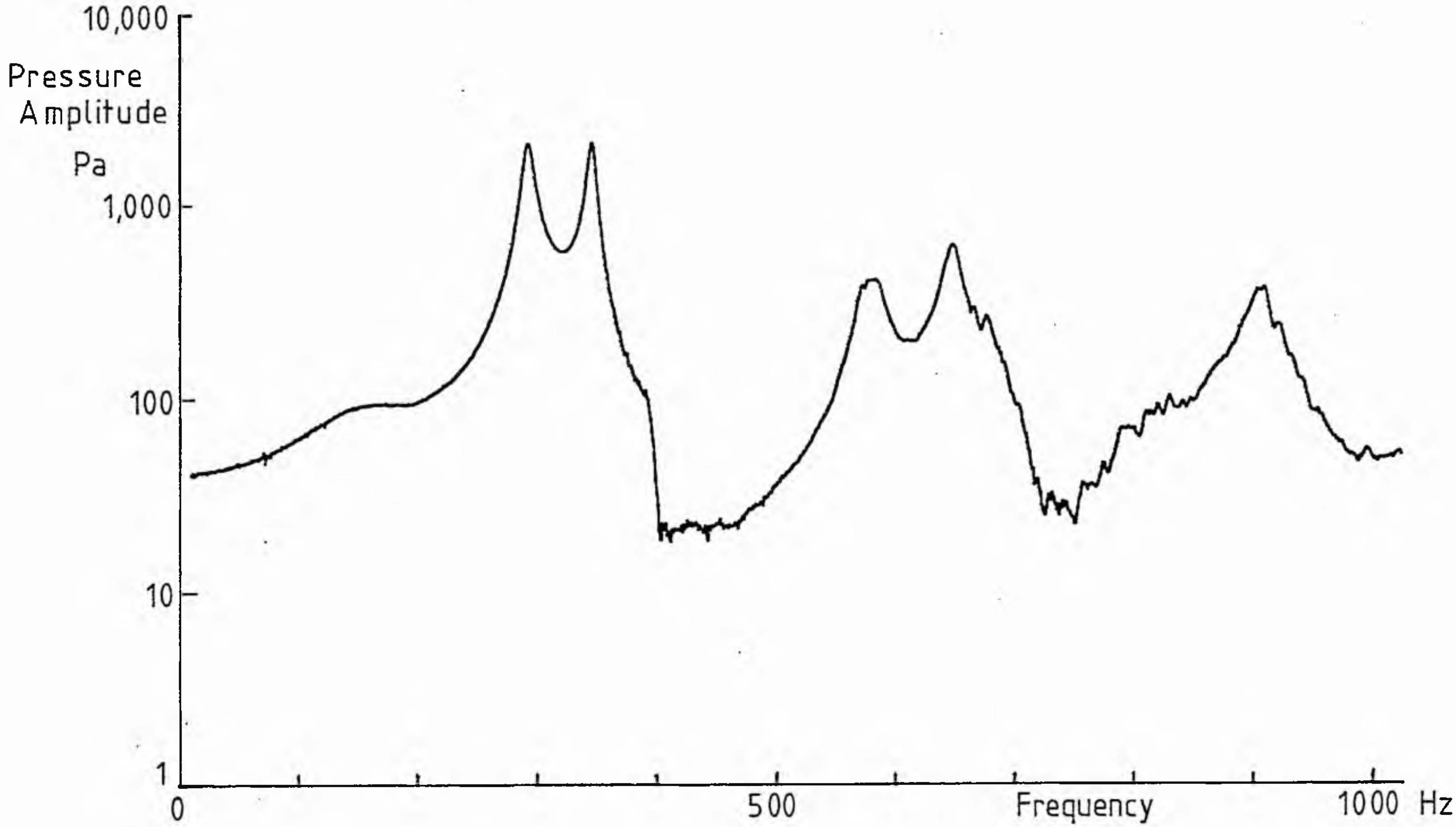

Figure 4.10 Pressure Amplitude at Bell Throat.

Figure (4.11) shows the general response for these cases. This result is obtained by omitting $f_r(x,y,z)$ from equation (4.3) such that the response shown is for no particular point, but is an indication of the general response of the bell. All the mode shapes are normalised to a maximum displacement of 1.0 so the relative heights of peaks are indicative of the relative responses of points with similar magnitudes of displacement in the normalised mode shapes. Also omitted is the forcing function shown in figure (4.10), so the curves in figure (4.11) can be considered as more of a mechanical mobility - frequency plot as opposed to, for example, a velocity - frequency plot.

It can be seen that there are a number of modes excited in the region of 200 Hz but not nearly as strongly as in the region of 450 Hz. Other less strongly excited modes can also be seen. As expected, the modes of the thicker bells have higher frequencies than those of the thinner, and that the thinner bells are more strongly excited than those of the thicker. These general observations are in agreement with the experimental findings.

Using realistic values for the excitation pressure amplitude and also using values for $f_r(x,y,z)$ in equation (4.3) at a node equivalent to the point of measurement of response (about 8 cm from the large end, node 276), the response of the bell was calculated and is shown in figure (4.12). The addition of these two parameters in the analyses has been sufficient to reduce the difference between the response of modes around 200 Hz and those around 450 Hz by several orders of magnitude, though unfortunately it has not been quite enough because no modes were measured corresponding to those around 450 Hz.

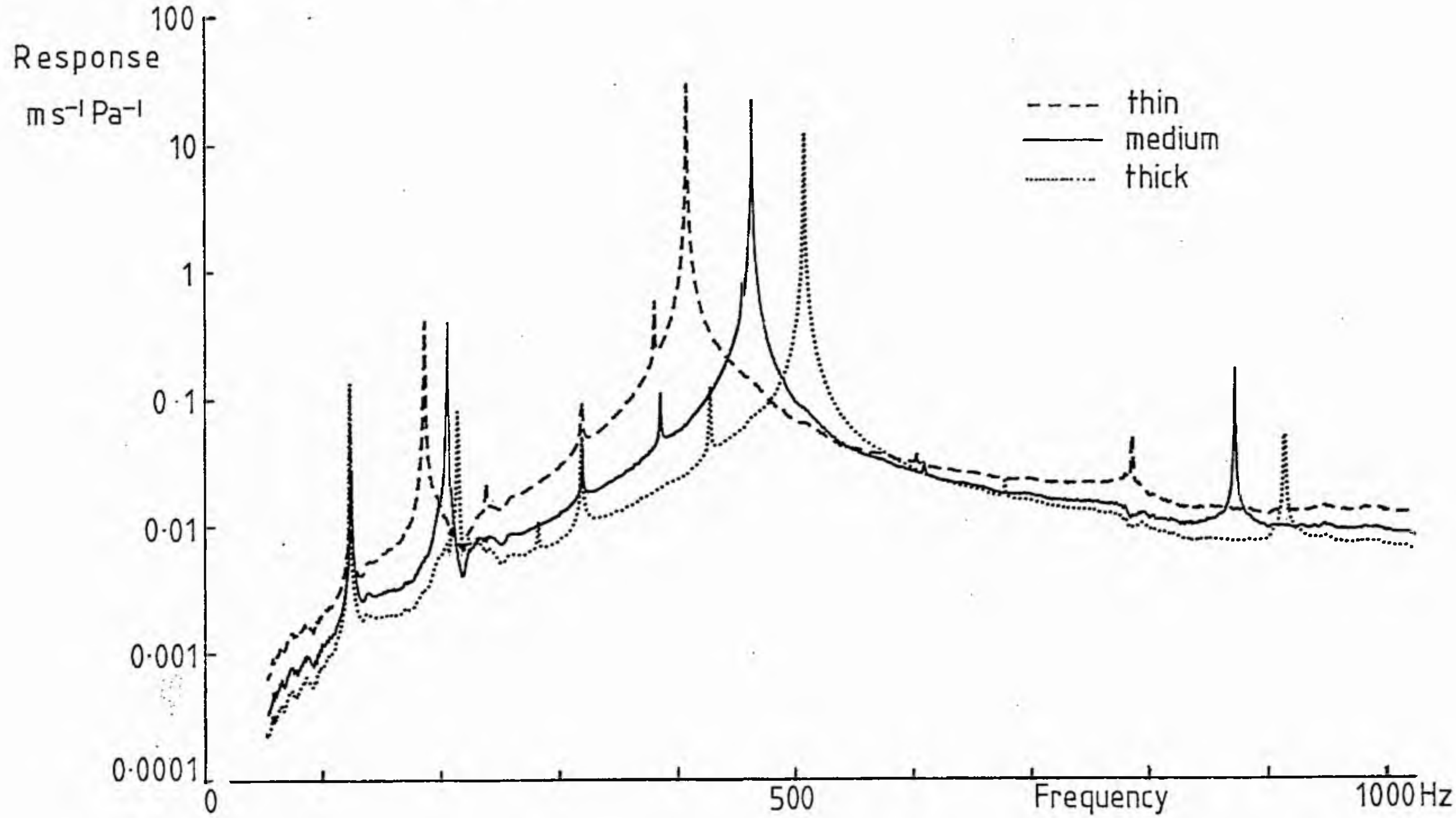

Figure 4.11 General Response of Trombone Bells Modelled for Comparison with Experimental Data.

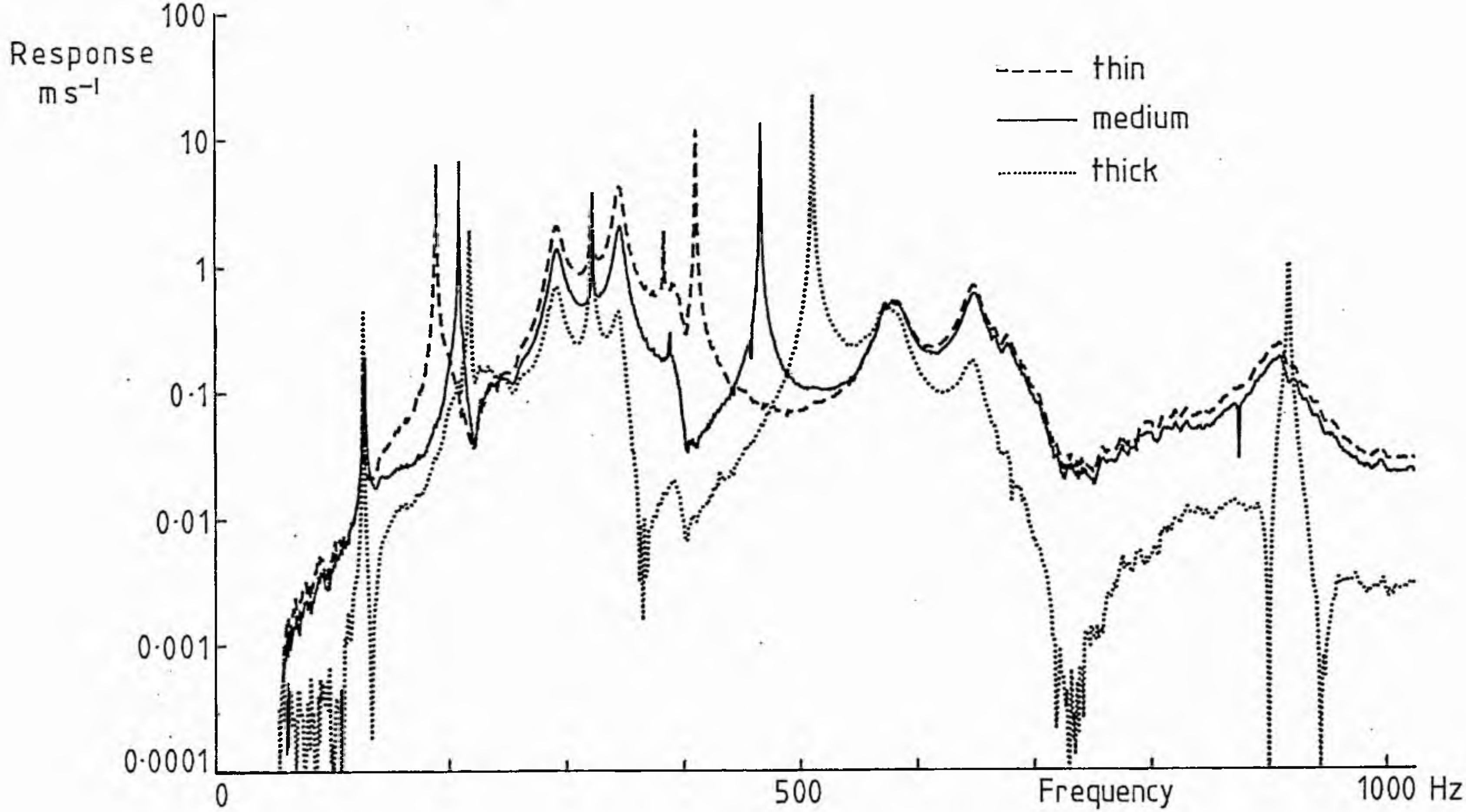

Figure 4.12 Calculated Response of a Point on Trombone Bells for Comparison with Experimental Data.

Figures (4.13), (4.14) and (4.15) show a comparison on an expanded frequency scale between the experimental and the calculated results. Agreement is mediocre but shows promise. A response is predicted rather than no response at all, and only a few modes are predicted to be strongly excited in the region of interest rather than many. A number of reasons could account for the disparities:

(a) Incorrect data for caculations. This is unlikely as numerous cases have been analysed fully other than those discussed in this chapter which have tested different combinations and intermediates of data values, none of which can justifiably either move the lower modes up sufficiently or move the higher modes down. Variations in rim size, material properties and other geometry would have to be changed too drastically to move the modes to fit the experimental data. Modelling the seams in the bell with thinner elements or introducing a small ovality into the undisturbed shape of the bell do not significantly change the mode frequencies or the degree of coupling to the driving force.

(b) Insufficient data. This is very likely as there could be properties of the bell which have not been fully described in the data submitted to the finite element package. A non-symmetrical bell shape or wall thickness could be enough to change the result of this section if the perturbation was large enough, though too small to be reliably measured during data collection. Also, the modelling of the cotton sling is crude but the best possible in the circumstances.

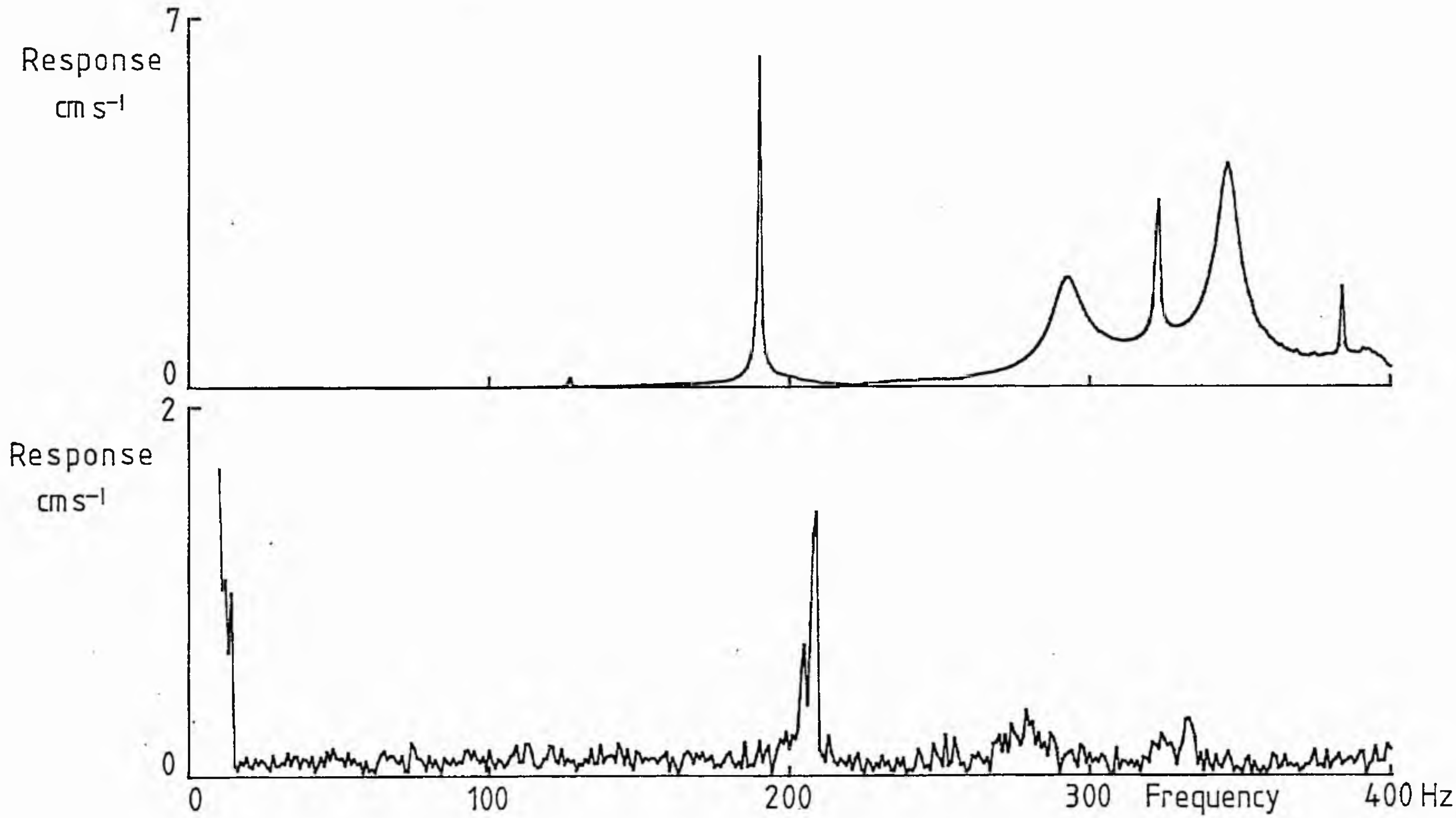

Figure 4.13 Response of Point on Thin Trombone Bell to Acoustic Excitation.

Upper Graph Calculated, Lower Graph Experimental.

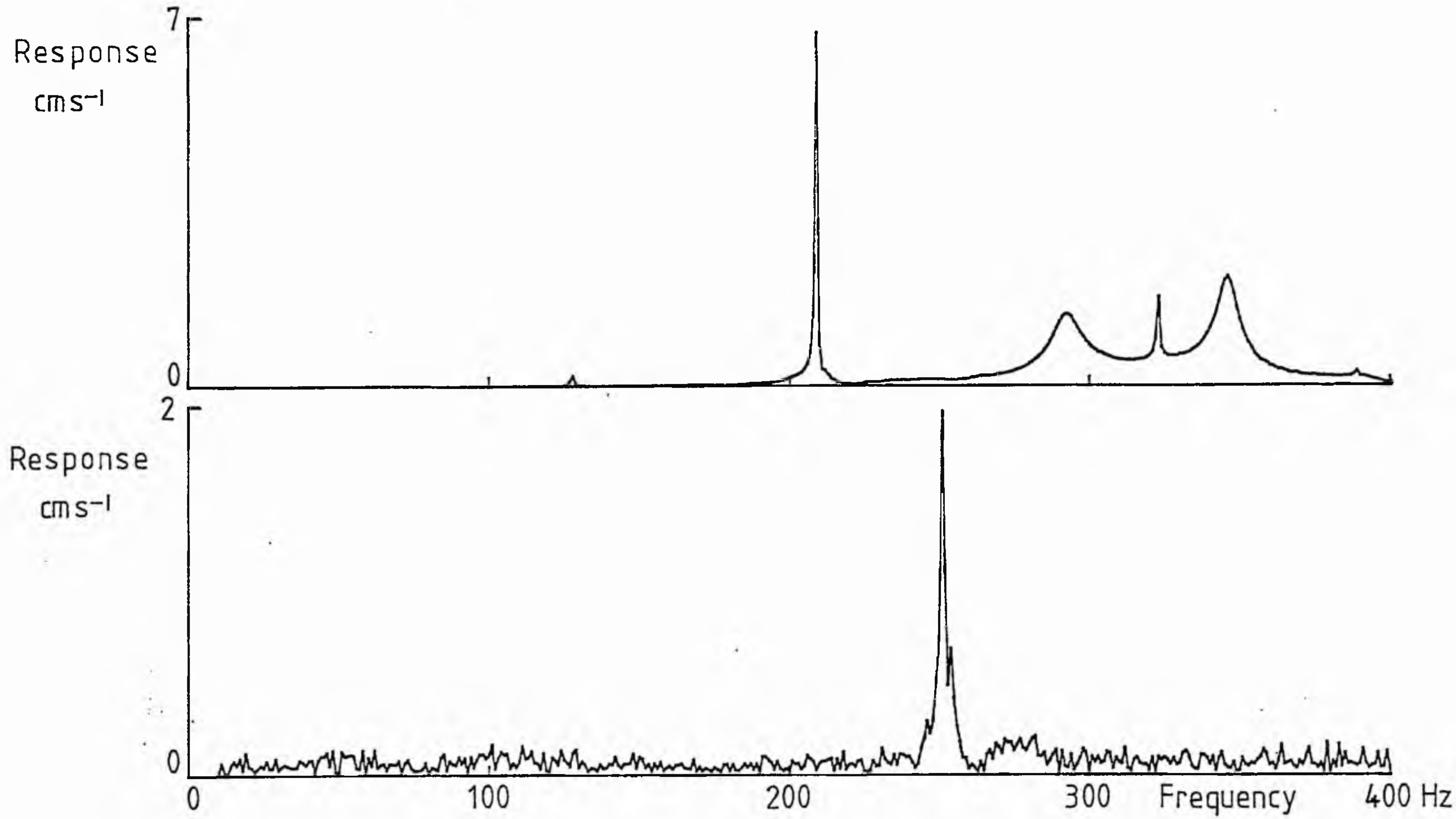

Figure 4.14 Response of Point on Medium Trombone Bell to Acoustic Excitation.

Upper Graph Calculated, Lower Graph Experimental.

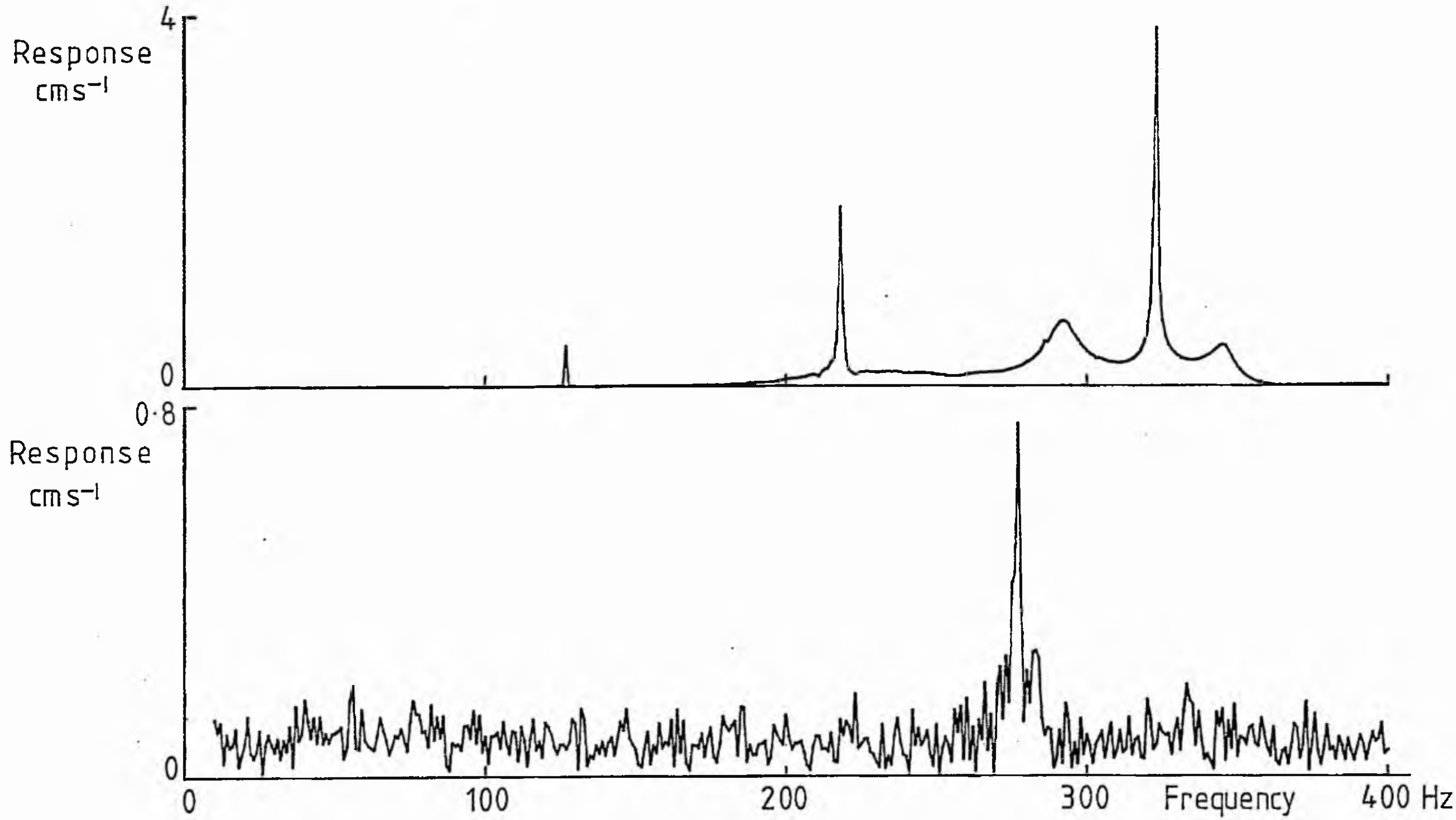

Figure 4.15 Response of Point on Thick Trombone Bell to Acoustic Excitation.

Upper Graph Calculated, Lower Graph Experimental.

(c) Incomplete and inappropriate method of analysis. The finite element analysis could be improved as outlined in section 4.2.2. The generalised vibration analysis could be improved by taking the rotations into account and thus including the excitation due to varying pressure gradient, and also improved values of damping are needed as these have a large effect on the predicted amplitude of excitation.

(d) Wrong experimental results. The main criticism of the experimental technique is the support of the bell by the cotton sling. This is the principal asymmetry which can be easily modelled in A.S.A.S. Other asymmetries, such as distortion due to gravity, cannot be modelled or are unknown. This implies in the calculations that the cotton sling is the property which aids in the excitation of modes, and thus plays a key part in the system rather than the negligible part it was intended to play.

These results are encouraging as it is only details in which the calculations and experiment disagree, in principle the agreement is good considering the crudeness of the analysis.

4.5.5 Effect of Bell Parameters

The data cases described in section 4.2.2 are a representative set drawn from the many cases analysed by the methods described in this chapter. The cases may be grouped into five sets, each investigating a particular parameter:

(a) Wall thickness: thin, medium and thick.

(b) Material: brass, silver, copper and nickel silver.

(c) Stay position: normal and nearer to the large end of the bell.

(d) Rim: normal, thick and non-existent.

(e) Geometric shape: Sovereign 937 bell and Imperial bell.

Also presented in this section is a very approximate result for the trumpet bells.

All the figures in this section are of the form of figure (4.11) viz. $f_r(x,y,z)$ is not included in equation (4.3) and the pressure amplitude is taken to be 1.0. Thus the curves represent a general response for the modes as opposed to a particular response of a point, and the values of response are a type of mobility (normally mobility × force = velocity, in this case "mobility" × pressure = velocity).

In all these cases the principal asymmetry and therefore major contributor to the excitation of modes (as explained in section 4.5.3) is the stay.

Figure (4.16) presents the results for bells of three different wall thicknesses; thin, medium and thick. Compared to the results for similar bells in section 4.5.4 the two noticeable differences are that the mode at 200 Hz for the medium bell is much more strongly excited than the equivalent mode for the other two bells, and that the higher frequency modes are much more strongly excited. Except at resonances it can be seen that the level of excitation is in bell thickness order, the thicker bell having the lower level of response. Also note that the resonances seem to be in three distinct frequency ranges.

The results for bells of different materials but identical geometry (the brass bell is the same as the medium bell in figure (4.16))

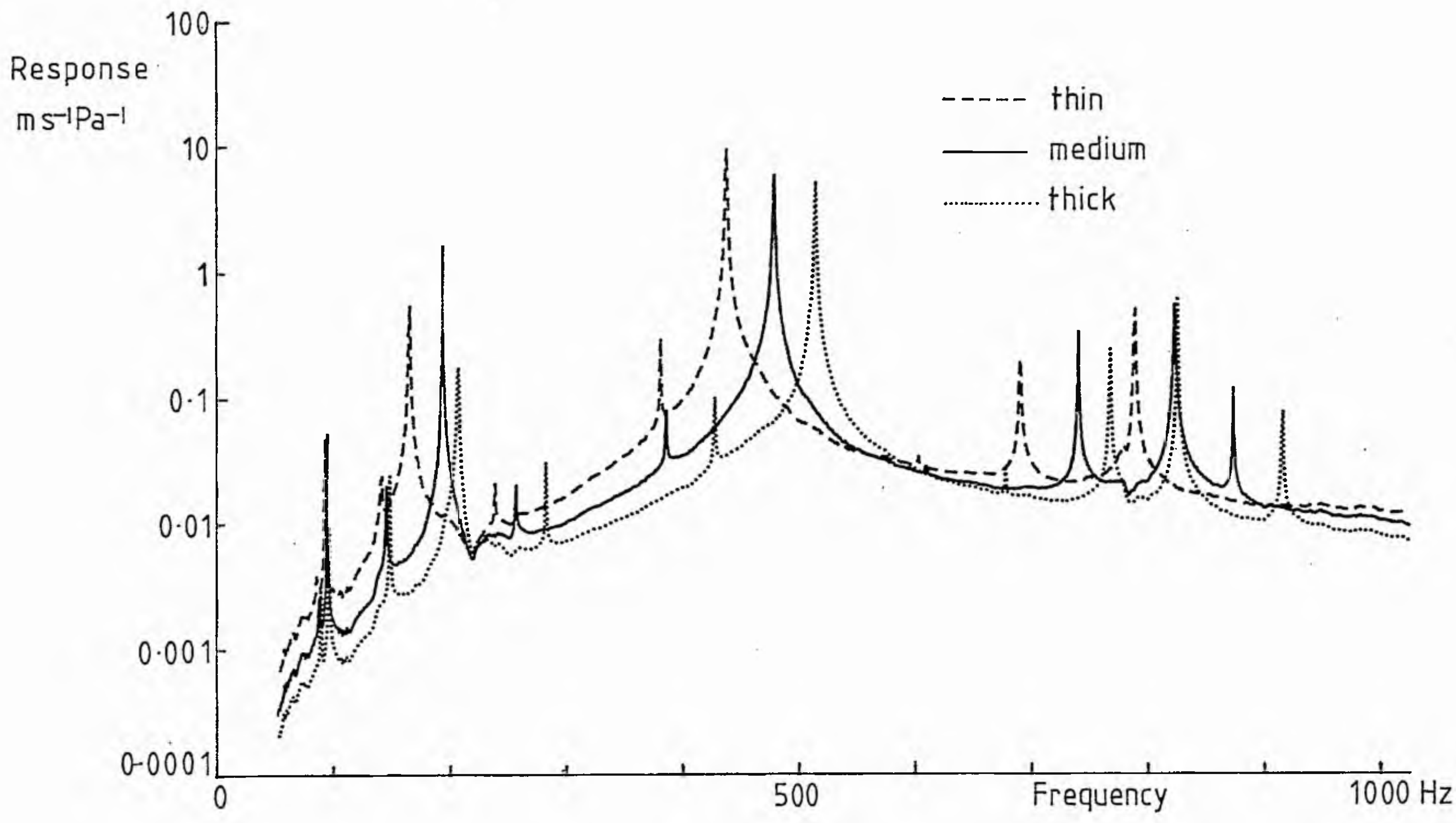

Figure 4.16 General Response of Trombone Bells of Different Thickness, Modelled as if Fitted on a Trombone.

are shown in figure (4.17). The result for the copper bell is not shown as it is so similar to that for brass; if it were plotted it would only thicken the solid curve. The responses of the brass and nickel silver bells are quite similar; the nickel silver bell having resonances at slightly higher frequency, particularly at high frequency. The response for the silver bell has a similar "signature" but the resonances are at much lower frequencies. It is this part of the analysis which would particularly benefit from a close study of contributions to damping, as the response of the modes where internal damping is significant will be particularly dependent on material. How significant internal damping is compared to radiation damping is unknown as so little is known about either when applied to these cases.

Figure (4.18) shows the effect of moving the stay. The frequencies of most, but not all, modes are increased and levels of response are generally lower when the stay is moved about 2.5 cm towards the large end of the bell.

The responses associated with bells having a normal rim, a thick rim and no rim at all are shown in figure (4.19). The two extreme cases have the effect of increasing response level, except for the thick rim at low frequency. As expected, the lower modes for the thick rim are of lower frequency than the normal and the higher modes are higher. For the bell with no rim, the opposite is the case; the lower modes are higher in frequency and the higher modes lower, if not lacking in quantity.

Figure (4.20) shows responses for the two different shapes of bell (Sovereign 937 and Imperial). The mode at around 500 Hz is higher in frequency and has a higher level of response for the Imperial bell than that of the Sovereign. The Sovereign bell, however, shows much more

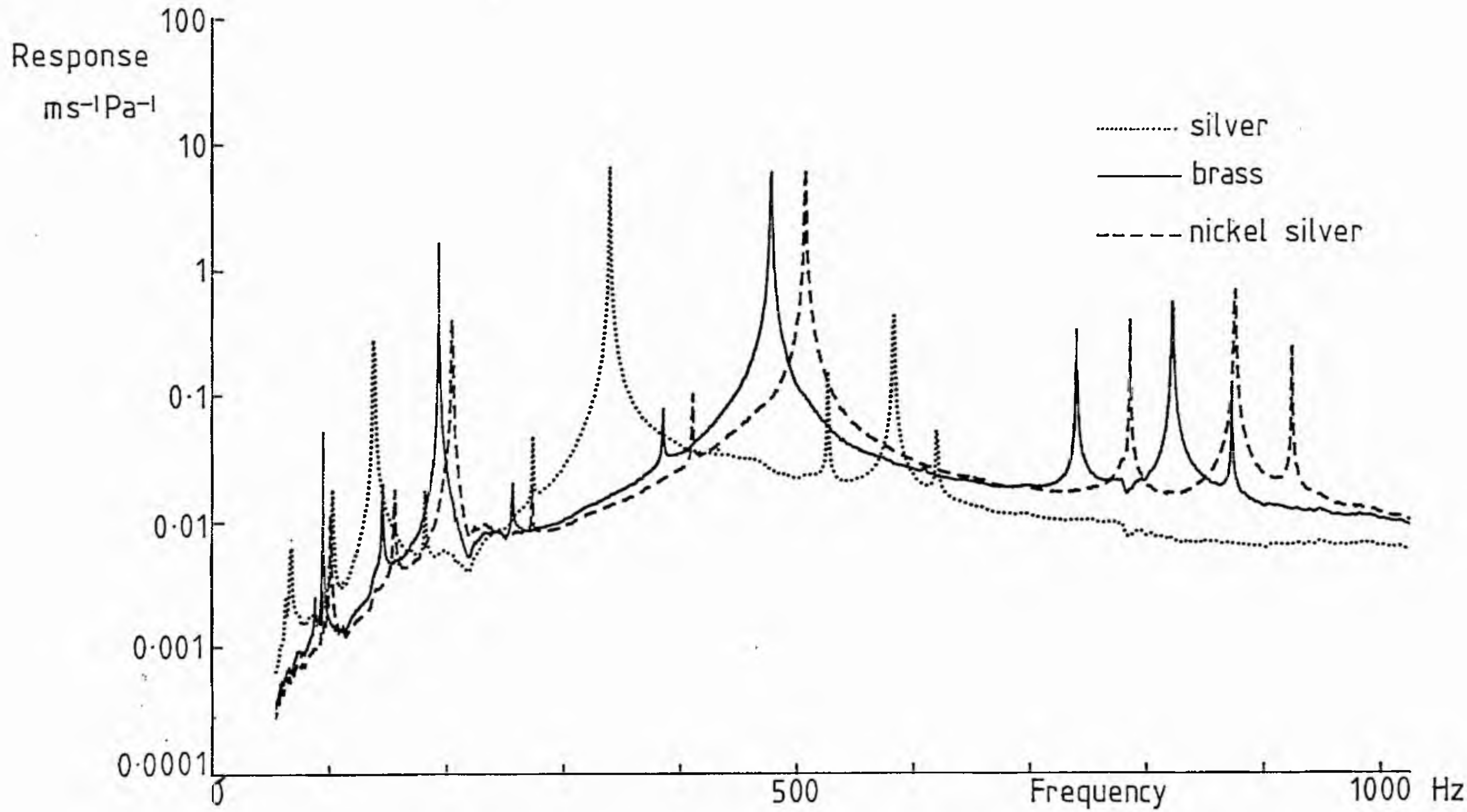

Figure 4.17 General Response of Trombone Bells of Different Material.

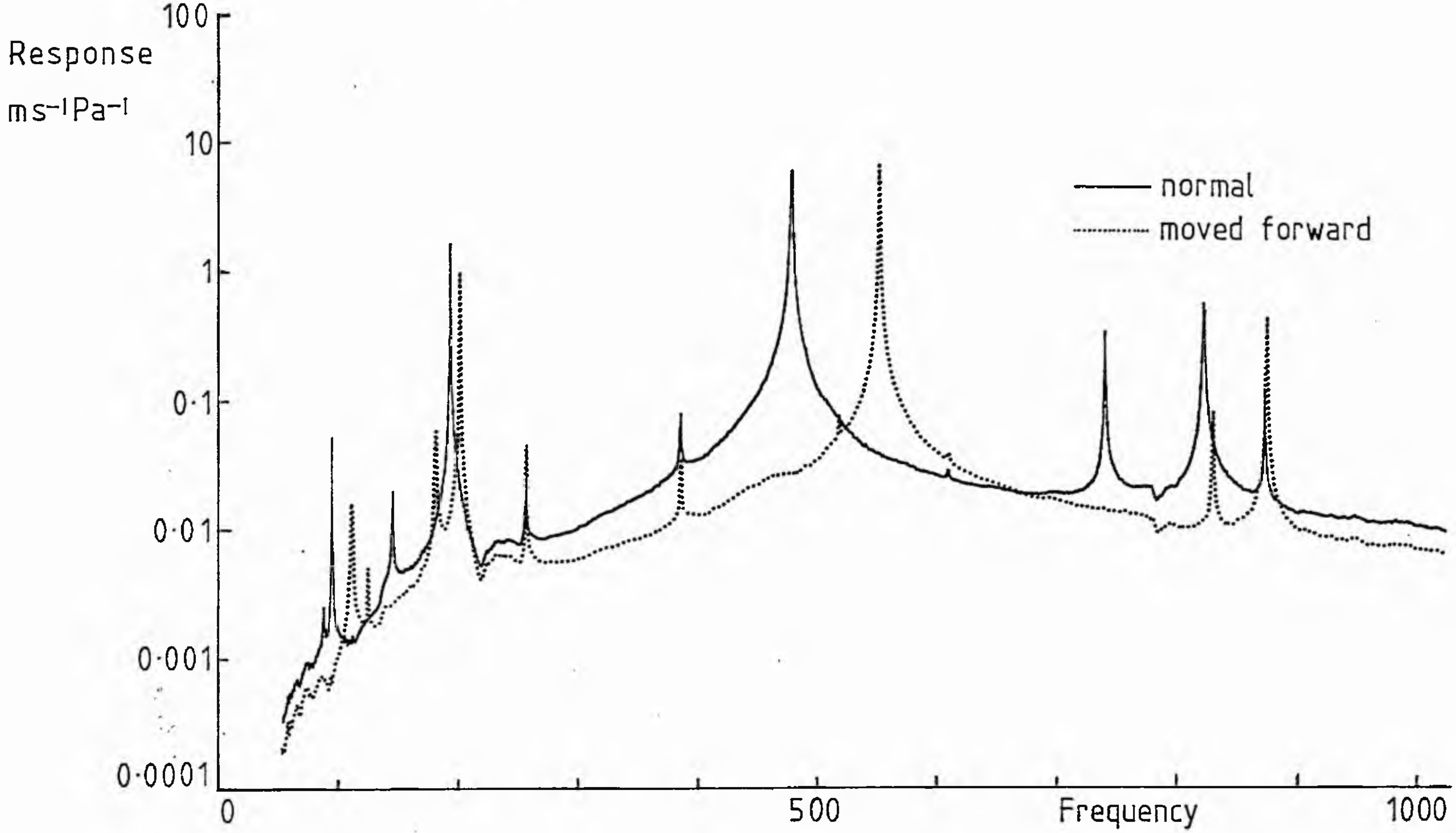

Figure 4.18 General Response of Trombone Bells with Different Stay Positions.

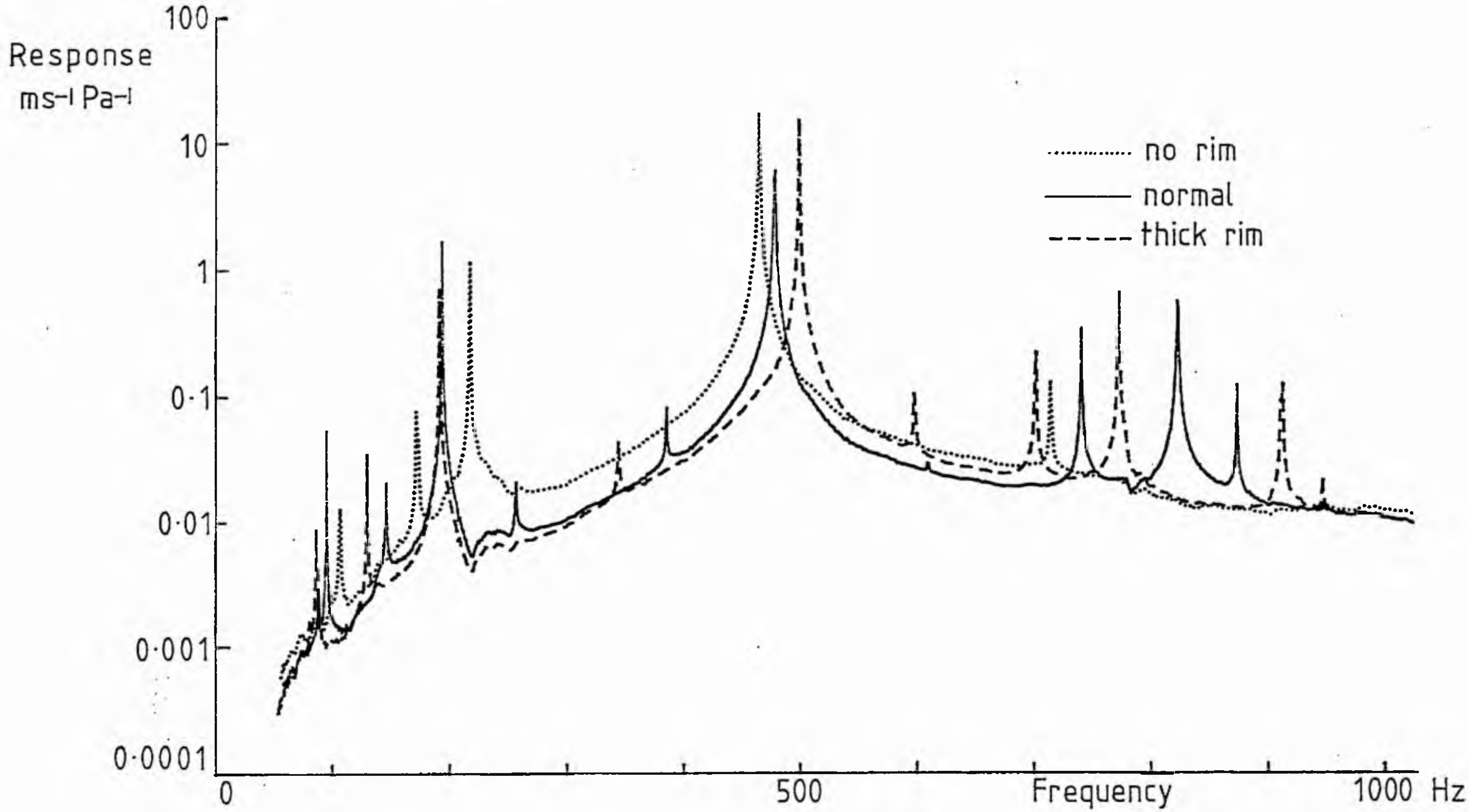

Figure 4.19 General Response of Trombone Bells with Different Rims.

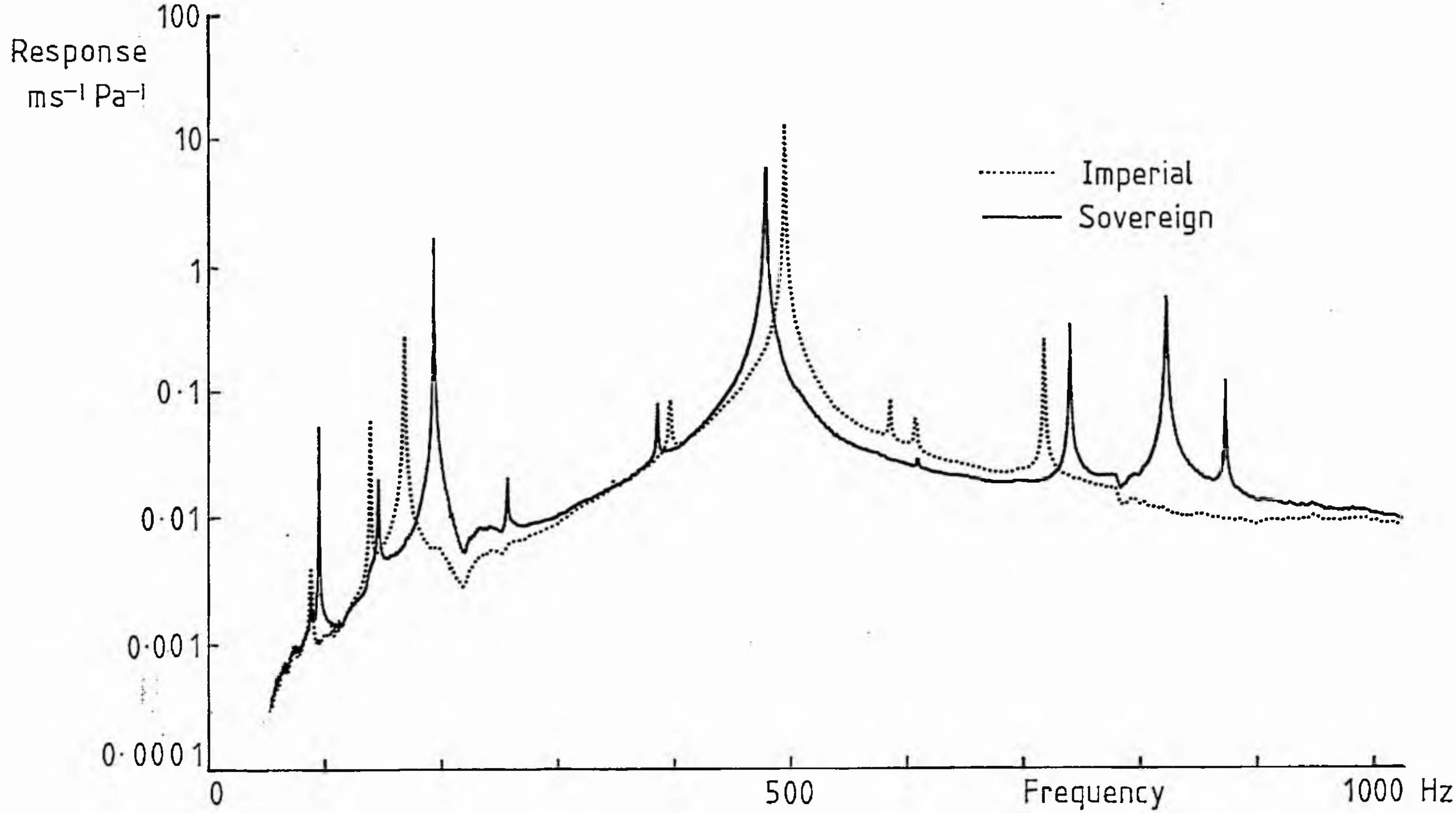

Figure 4.20 General Response of Trombone Bells with Different Geometric Shape.

resonant activity in both the high frequency and low frequency groups of modes.

For purposes of comparison, the output from the finite element analyses of the trumpet bells were processed similarly to the trombone bells. To get these results, the pressure distribution for the trombone bells were used but assumed to be for twice the frequency at which they were measured. This is very approximate but will produce results with which the results for the two bells may be compared.

The responses for the trumpet bells are shown in figure (4.21). Except for the levels of two resonances, the thicker bell shows a higher level of excitation and slightly higher resonant frequencies. Other than that, the two are very similar.

All these cases above show that differences in frequencies of the natural modes is the major distinction between cases. Overall levels of response do not vary much though this may be different if more was known about the nature of the damping mechanisms. All cases show a very similar pattern of response, which coincides with the result from section 4.4.1 that the set of mode shapes illustrated are typical of all the data cases analysed.

4.5.6 Summary

The finite element package has been used to produce the frequencies and shapes of the natural modes for several trombone bells and two trumpet bells. Commercially available software interfaced with software written by the author has been utilised to display these mode shapes in perspective.

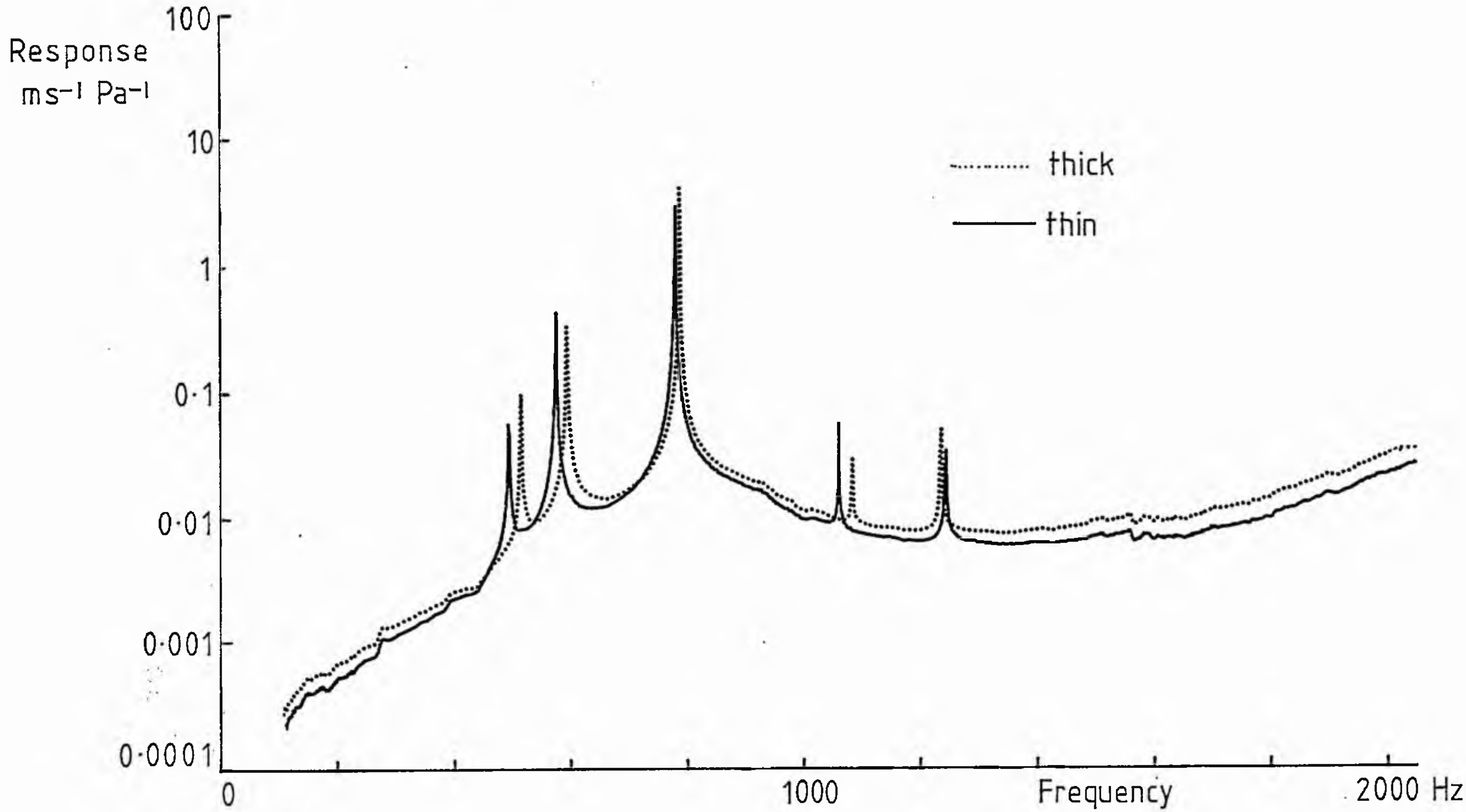

Figure 4.21 General Response of Trumpet Bells of Different Wall Thickness.

A general method for vibration analysis is described and implemented to be applied to the output from the finite element package. This method is used to generate results for comparison with experimental results, there being agreement in general but not in detail. The method is also used to compare the effects of changing major properties of the bell, both material and geometric, showing that the general responses are similar, the major difference being the frequencies of the resonances.

Listings of programs described in this chapter are in appendix D.

CHAPTER 5 : DISCUSSION AND CONCLUSIONS

5.1 Summary

The first purpose of this thesis is to formalise the extent to which the walls of brass instruments can have musical effect, and to formalise the logical ways in which they could have effect. It is shown in Chapter 2 how the manufacturers and players view the subject, as well as the scientists. Also highlighted is the lack of good scientific study of the subject; the investigations which have been performed are very inconclusive and are not fundamental.

The many ways in which the instrument walls could determine the characteristic of an instrument are discussed and catalogued in section 2.4 so that scientific experiments may be devised to test the significance of the various contributions.

The second purpose of this thesis is to report the experiments devised to contribute to the understanding of some of the effects discussed in Chapter 2.

The first set of experiments investigate the properties of internal surface finish of tubes as applied to brass instruments. The obvious scientific measure to be applied in this case is that of sound attenuation. A player interacts with a standing wave in an instrument and the characteristic of this standing wave is affected by the attenuation in the tubing. A novel and accurate method for deducing attenuation is described in Chapter 3: a method which does not rely on the insertion of long probes along the length of the tube, which themselves contribute to the attenuation of the system. The method is particularly suited to tubes of the size found, for example, in trombones (diameter about 1 cm).

Measurements are made on a length of cylindrical brass tubing which has been treated in several ways to provide a variety of internal surface finish typical of finishes found in brass instruments, plus one extreme test case of high attenuation. The tubes with smooth, slightly roughened and thinly, evenly coated finishes showed only small variations in attenuation. The significant increase in attenuation is shown by the tube with an internal coating of coarse peanut paste; this simulates an extreme case where an instrument has not been cleaned for a long time.

This result has prompted a set of subjective experiments (Shepherd 1980) where six identical trombones were tested on several players, three trombones had the first section of tubing internally coated with coarse peanut paste. The players under test could reliably distinguish between the trombones with coating and the ones without, the preference being the trombones without the coating. As trivial as this result may sound, it is a definite link between subjective and objective measurements, from which future investigation may be prompted and refined. Much of the subjective and objective work to date has shown little interconnection.

The result for the roughened tube shows no significant overall increase in attenuation but does show much more inconsistency, so there could be a turbulence problem which would affect the player but not the measuring system; the measuring system has much better discrimination against noisy signals than a brass player.

The aim is to be able to measure reliable differences in instruments which players can also distinguish under playing and experimental conditions. It has been shown, for example, that an increase of attenuation of the magnitude shown by figure (3.14) can be recognised by a player; but is the measure sensitive enough? Can a player distinguish between attenuations shown by figures (3.10) and (3.13)? If so, then it can make the difference between a manufacturer carefully polishing the bore of an instrument (if that be the desired finish) or not.

Extending the method for cylindrical tubes to actual instruments is not easy. The theory breaks down for non-cylindrical tubing and is not easily adjusted to cater for even the mildly flaring portions of brass instruments. Much work is needed with equations (B12) and (B13) to produce a solution in a convenient form for representing a brass instrument.

A further difficulty is that of the numerous abrupt discontinuities along the length of an instrument e.g. where tubing is joined, or at the ends of slides. Also, for example, in a dirty instrument the amount of sediment will decrease along the length of the instrument causing another drift from the ideal case.

Generally, it can be concluded that it is sediment and dirt in an instrument that causes the most significant increase in attenuation. Players can distinguish between clean and dirty (but otherwise identical) instruments and prefer a clean one. However, if a player continuously plays an instrument, as it becomes internally soiled he will be adjusting his own playing to compensate and will perhaps not notice any difference.

The second part of this thesis investigates the vibration properties of brass instrument bells, the principal example being that of the trombone.

Finite element techniques are used to extract the natural mode frequencies and shapes of an otherwise intractable structure. The mode shapes are displayed in perspective and show a variety of axial and circumferential wavelengths. A set of mode shapes is also presented for a trumpet bell, these modes being less obvious in shape than those for the trombone. The mode shape presented by Smith (1978) can be recognised in figure (4.6), either F or G which corresponds to his frequency of 250 Hz.

A variety of data sets investigating the effect of various trombone bell features have been analysed, the most significant of which are presented in Chapter 4. A method of generalised vibration analysis is described and used on the results of these data sets to compare levels of excitation. The difficulty throughout these analyses is a lack of knowledge of damping. Experimental values are used though are not necessarily applicable, for example, over the range of materials used. The magnitudes of response do not vary greatly from case to case, but the frequencies of resonances do.

If these resonances were to have any musical effect, it will be only at specific notes rather than of a musical range of frequencies so will their effect be good or bad? A possible analogy could be the "wolf note" on a violin, but this latter effect is not nearly so widely acclaimed for brass instruments.

It is thought that the resonances of the bell structure may reinforce the higher notes of the instrument. This is possible but could be better investigated with more modes extracted, for this better elements and a finer mesh would be needed.

Some work by Smith (1980) has shown that for different thickness trombone bells on otherwise identical instruments, there is a difference in the harmonic structure of the sound reaching the player's ear, this difference possibly being attributed to the sound radiated from the bell surface. Although the theoretical work of this thesis has not been taken far enough to approach this result it is encouraging to have a result to work towards. Much more work is needed on the radiation from the bell surface to couple the vibration properties discussed in Chapter 4 to a theoretically deduced sound field at a point corresponding to a trombone player's ear.

An attempt is made to achieve results comparable to the work of Kitchin (1980). In hindsight it would have been better to have supported his trombone bells rigidly, similar to when mounted on a trombone, rather than loosely with a cotton thread; the latter being difficult to model with the finite element package used. However, considering the crudeness of the model and analysis technique, the result that the calculations show a similar form of result to the experiment is gratifying. The disagreement in detail is annoying but with further refinements hopefully rectifiable.

An important implication of the vibration analysis is that a mode is excited in proportion to its degree of asymmetry. A perfectly symmetric mode will yield zero coupling to the internal forcing pressure when equation (4.9) is evaluated. This implies that it is those properties of the bell which lead to asymmetric mode shapes that are either excited or not. On the trombone the major contributor is the stay but other parts such as the seam or non-circular cross sectional bell shape may also be formative in practice.

The frequencies of the modes are dominated by material of construction, overall bell shape and thickness, and nature of the rim: all of which are found in a variety of sizes, shapes and constituency on trombones.

The musical significance of these properties is a difficult deduction. When questions such as: what is the desired vibration characteristic of the bell?, has no proven answer, one must keep an open mind and not dismiss superficially trivial matters. For example, the result for the change in stay position could contribute to the difference between instruments with or without the Bb/F valve, the instruments with this valve fitted having a different stay position.

Should manufacturers make more of an effort to make their bells more axisymmetric; or should they be deliberately elliptical?

Generally it can be concluded that the material and geometric properties of the bell as discussed above have major effects on their vibration properties. The significance of these properties is not known, though some results show that there may be significance to be found. Much more data is required, particularly for values of damping and also more detailed measurements of bell construction. These, linked with a more thorough and accurate analysis technique, possibly using finite elements to model the air inside and outside the trombone, could yield some very important results and resolve many of the current arguments as to the musical contribution of the bell structure.

5.2 Conclusions

(a) There is vehement disagreement between manufacturers, players and scientists as to the contribution made by the wall properties to the musical quality of brass instruments.

(b) Scientific literature on the subject is sparse, and that which exists does not make a good foundation on which to base further investigations.

(c) The ways in which walls can influence the quality of an instrument can be grouped under four headings: visual, palpable, direct aural and physical acoustic.

(d) The impedance/transfer function method is an accurate method for deducing attenuation in tubes of diameter approximately 1 cm over the frequency range applicable to the trombone.

(e) Clean, evenly thinly coated and internally roughened tubes show similar levels of attenuation, whereas a tube with a heavy and coarse internal coating shows a significant increase in attenuation.

(f) This increase in attenuation can be distinguished by the measuring system and by players when instruments are treated in a similar manner.

(g) The measurement technique is not easily extended to measurements on actual instruments.

(h) The finite element method is the best method for vibration analysis of brass instrument bells because of the ready availability of commercial computer packages and the mathematical intractability of the bell shape.

(i) The properties of the bell which have the most significant effect on the frequencies of the natural vibration modes are wall thickness, material, stay position, rim size and geometric shape. (not in order of importance).

(j) Predictions of levels of response of bells to acoustic excitation is difficult due to the lack of data of damping, both internal and radiation. There is a general agreement with some experimental results.

(k) The musical significance of the bell vibration characteristics is not known. Experiments indicate that future work should concentrate on sound radiation from the bell surface and the sound field at the players ear for different types of bell.

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APPENDIX A

Manufacturer's Claims

In the brass instrument market, a very significant feature is the advertising machine. Manufacturers publish catalogues telling the player what makes a good instrument, and why he should choose a particular instrument by describing its properties; a situation which leads to the creation of a grand mixture of fact, folklore, repute, myth and mystique.

Despite the general uncertainty as to the rôle material plays on the musical properties of an instrument, many manufacturers are (according to their catalogues and other published information) quite confident in their claims on the matter. The following paragraphs are a review of the subject. All phrases in quotation marks are taken from sales catalogues and information documents provided by brass instrument manufacturers: Bach, Mirafone, Yamaha, Schilke, Paxman, Martin, Holton, Reynolds, Olds, King, Getzen, and Conn. No conscious attempt has been made to alter the emphasis made by manufacturers though many quotes have been slightly edited to remove irrelevant asides and to provide continuity.

In the preamble of his catalogue, Bach states that "theoretically the shape, not the material, of an instrument determines its timbre. Yet sensitive players insist that alloys in brass instruments are as distinctive in sound as in appearance", yet later is more committed by claiming that ".... bells of yellow brass .... produce brilliant, lively tone". Mirafone can provide you with an instrument made with a "special formula yellow brass bell for that big, yet centered tone". Yamaha support Bach with "the adoption of bright sounding yellow brass .... bell material...." and tell you that it "provides the instrument with high resonance and superb sonority", though at a slightly higher price they can sell you an instrument ".... with beautiful resonance, easy to stress, bright timbre thanks to yellow brass two piece pounded bell". However, Schilke contradicts this by stating ".... that with a yellow brass, it is necessary after the bell is formed to anneal it at two different points .... if this temper were left in the bell, we would find the quality of sound had become very dark". Alternatively, Paxman say that " .... yellow brass makes for warm sonority ....".

Many manufacturers quote material (usually bell material) in their sales specifications; Martin specify "natural yellow brass", Holton "regular (yellow) brass", Yamaha talk generally about their "special Yamaha alloys" then specify bell alloy for trumpets, cornets, flugelhorns, and trombones, yet not for horns, euphoniums, or basses?

On the subject of the French horn, Reynolds tell you that "use of brass rather than nickel silver gives the instrument a somewhat brighter freer tone", and Conn agree by saying that "use of a brass bell rather than nickel silver gives this instrument a slightly brighter tone".

This is explained when Reynolds reveal that "solid nickel silver construction produces a dark resonant sound", but Paxman cast doubt by claiming that "a considerably 'darkened' tone quality was attributed to the alloy (wrongly, we think) rather than to a radically different bell profile .... nickel silver brightens and .... condenses the tone into a crisp clarity".

A Reynolds trombone is "nickel plated for contemporary apperance and sound", or for the tuba player Conn say that "hard nickel plate on the bell flare gives better resonance". Schilke, on the other hand, claims that " .... plating does not affect the playing qualities of brass instruments", though this does not deter Olds from praising their Superstar range of which "the bright silver finish projects a quality of sound that stands out and takes the lead". King produce a cornet with a "solid sterling silver bell.... projects tone farther with less effort".

Though yellow brass tends to be the standard material, many top range models include gold (or red) brass sections as standard or as an alternative. Between Conn, Bach, Olds, Paxman, and Mirafone, this material destines the sound to be dark, veiled, mellow, sonorous, resilient, compact, round, full bodied, delicious, and rich, and gives the sound greater carrying power. These claims are reasonably consistent compared to those about instruments made from other materials and processes, though they still have dubious substantiation. Holton, Yamaha, Getzen, and Reynolds all specify gold (or red) brass for some of their instruments though say little or nothing about it.

Other miscellaneous materials are used; Conn produce "Coprion" bells electroformed from copper which on a trumpet gives a "darker richer tone". Conn also produce an "Electro-D" bell which is electroformed from a non-specified alloy, but it is specified that the "Electro-D" bell adds stability and precision of response".

Reynolds produce a trombone on which the "Bronze-O-Lite bell flare centres and delivers the tone", and another on which the "bronze alloy bell gives .... a richer, darker sound and excellent projection".

Schilke found that by ".... tapping the steel bell, it would emit a very ringing sound. However, when we played this instrument, the quality of the sound was extremely dead .... the lead bell .... if rapped .... emitted an extremely dead sound like rapping a piece of wood. However, the sound that emanated when it was blown was extremely brilliant". His findings led him to use his "beryllium bronze" of which he says "this particular material has a wonderful acoustical effect in that it has remarkable carrying power. Its projection of sound is quite phenomenal".

Mirafone use a "secret formula brass. As resonant as it is flawless, this secret formula brass produces the symphony sound which distinguishes Mirafone from all other brass instruments".

In many cases it is not just the alloy which is allocated specific properties but also its thickness and mechanical state. We have already met pounded bells and annealed bells, but Conn maintian that "electroforming offers control over the shape, grain structure, and metal thickness .... bells produced by this method have outstanding tonal qualities over a wide dynamic range". Alternatively, in the Yamaha factory "the bell is pounded, greatly enhancing the instruments good response and unique tonal beauty". Olds explain the principle of their "ultrasonic bells", which are seamless because seams "really show up in the sound. The acoustically 'dead' solder seams break up the horn's natural vibrations, or sound waves .... just as a breakwater prevents the natural flow of sea waves". The seamless theme is also hailed by King whose "seamless bell tail is wave-free and uniform in grain structure and metal thickness", and Yamaha whose "bell, seamless welded, non engraved, formed with varied wall thickness, provides superb tonal quality, precise intonation, and even response in all registers". Despite the Olds "ultrasonic bell", the Old's trombone with "the remarkable 'tone band', positioned on the bell flare, provides the timbre required in contemporary music", and Yamaha mention that "a special leaded bell rim enhances the instrument's good response and well-centered tonality".

Some people rate instruments by tapping the bell; Martin's 'Urbie Green' trombone goes "ding instead of clunk. This instrument vibrates when you play .... you can actually feel the note .... it's alive". Reynolds produce a trumpet with a ".... live bell ....", and Getzen's Eterna trumpet has a "bell with special acoustic properties and resonance that you can feel as well as hear". On the other hand, Schilke says that "as far as the overall instrument is concerned, the more inert it is to vibration, the better it is. However the thickness of the metal and the temper of metal in the mouthpiece, tuning slide, and bell greatly affect the quality of sound produced by the instrument", also that "metal with excess temper gives too many vibrations of its own".

Wall thickness is an oft mentioned property, Bach says that "for maximum resonance, one piece bells are graduated in thickness according to a secret formula", and produces an instrument "with a thin gauge bell for brilliance and minimum resistance". Yamaha "bells are precision formed with automatically controlled thickness .... micrometer tested for unfailing strength and tonal response", but they seem unsure as to what that thickness should be: they make instruments with a "bell that has been formed and worked to uniform thickness for producing superior tonal quality", and instruments where "there is a subtle variance in bell wall thickness which critically affects the instruments tonality". Schilke talks about "the thickness of an ordinary brass bell, I like the vulnerable areas to be .... between 12 and 14 thou .... with beryllium bronze, I am able to make bells with vulnerable areas to 6 and 7 thou", which contradicts what he said about instruments having to be as inert to vibration as possible.

It is quite widely accepted that the lacquer used on violins can critically affect acoustic properties of the instrument. To some, the same is felt about brass instruments. Conn use a finish called "Lustre-Conn" of which "only a very thin coating is needed, preserving the natural beauty and tone of the instrument". Yamaha claim that ".... our advances in chemistry have created finishes that result in the highest sound clarity, detail and realism", but Schilke says that his "lacquered instrument .... had a very much impaired tonal quality and the overall pitch was changed" which, he says, is due to the fact that lacquer is ≃ 7 thou thick whereas a finish like silver is only ≃ ½ thou thick (which he claims does not affect the instrument).

Most of the above material is exclusively directed at the bell of the instrument, though we have already had Schilke on the wall thickness of the mouthpipe and tuning slide. Additional to the solid sterling silver bell on King cornets is a "solid sterling silver mouthpipe ...." which ".... utilises the unique properties of sterling silver to provide a dark, but extremely resonant sound", and Yamaha make a trombone on which "the integral-drawn, yellow brass one-piece outer slide .... contributes to this instrument's good resonance".

As can be seen, there is much written about the mechanical and physical properties of materials and finishes used on brass instruments. Manufacturers seem to be in little doubt that materials play a vital rôle on the quality of acoustics of an instrument yet they contradict each other when it comes to identifying this role. Schilke is the only one who offers any evidence to back up claims, but his experiments are far from rigorous and his explanations far from scientific.

APPENDIX B

Aspects of Transmission Line Theory

B.1. Basic Theory

When considering the propagation of sound in a uniform waveguide, it is convenient to describe the nature of this propagation in terms of the transmission line parameters. There are two limitations to the use of this theory:

(a) The system must be linear.

(b) The mode of propagation must be a plane wave, i.e. the wavelength of sound must be much greater than the dimensions of cross section of the waveguide.

Assumption (a) breaks down if the sound pressure level becomes too high, or if turbulence is induced.

Assumption (b) limits the application to cylindrical tubes of up to about 3 cm diameter at 1 kHz, or frequencies up to 3 kHz for a tube of 1 cm diameter.

Expositions of transmission line theory can be found in Connor (1972) or to a greater complexity in Slater (1942).

Referring to figure (B.1), the equations that follow derive from the solutions to the equations:-

$$\frac{d^2P}{dx^2} = \gamma^2 P \qquad (B.1)$$

$$\frac{d^2U}{dx^2} = \gamma^2 U \qquad (B.2)$$

where $\gamma = \sqrt{Z.Y}$

is known as the propagation coefficient and is complex.

Solutions to (B.1) and (B.2) are of the standard form:-

$$P(x) = A\ e^{-\gamma x} + B\ e^{\gamma x} \qquad (B.3)$$

$$U(x) = \frac{A}{Z_c}\ e^{-\gamma x} - \frac{B}{Z_c}\ e^{\gamma x} \qquad (B.4)$$

The reflection coefficient at the termination of a tube of length ℓ m is defined as:-

$$R_\ell = \frac{B\ e^{\gamma\ell}}{A\ e^{-\gamma\ell}} \qquad (B.5)$$

it is easily shown from (B.3), (B.4) and (B.5) that

$$R_\ell = \frac{Z_\ell - Z_c}{Z_\ell + Z_c} \qquad (B.6)$$

where $Z_\ell = \frac{P_\ell}{U_\ell}$

B.2. Derived Useful Formulae

Input impedance:- $Z_0 = \frac{P_0}{U_0}$

from B.3, B.4 and B.5

$$Z_0 = Z_c \cdot \frac{(1+R_\ell e^{-2\gamma\ell})}{(1-R_\ell e^{-2\gamma\ell})} \qquad (B.7)$$

Transfer function:- $T = \frac{P_\ell}{P_0}$

from (B.3), (B.4) and (B.5)

$$T = \frac{e^{-\gamma\ell}(1+R_\ell)}{(1+R_\ell e^{-2\gamma\ell})} \qquad (B.8)$$

Propagation Coefficient:-

defining $R_0 = \frac{Z_0 - Z_c}{Z_0 + Z_c}$

and knowing that $\frac{R_0}{R_\ell} = e^{-2\gamma\ell}$ (B.9)

then from (B.9) and (B.8) the following quadratic can be derived:-

$$R_0 . e^{2\gamma\ell} - T(1 + R_0).e^{\gamma\ell} + 1 = 0 \qquad (B.10)$$

which can be solved for $e^{\gamma\ell}$.

Termination reflection coefficient:-

from (B.8) and (B.9) it can be shown that

$$R_0 . R_\ell^2 + (2R_0 - T^2(1 + R_0)).R_\ell + R_0 = 0 \qquad (B.11)$$

which can be solved for R_ℓ.

B.3. Extension to Line of Continuously Varying Parameters

For this case, the differential equations are (Slater 1942)

$$\frac{d^2P}{dx^2} - \frac{d \ln Z}{dx} . \frac{dP}{dx} - YZP = 0 \qquad (B.12)$$

$$\frac{d^2U}{dx^2} - \frac{d \ln Y}{dx} . \frac{dU}{dx} - YZU = 0 \qquad (B.13)$$

Y and Z are now functions of x and the solution to (B.12) and (B.13) are dependent on these functions.

For Y, Z = f(x) = constant then (B.12) and (B.13) become (B.1) and (B.2.)

For Y and Z being slowly varying functions of x then Slater (1942) derives an approximate solution:-

$$P(x) = A\sqrt{Z_c}e^{-\int \gamma dx} + B\sqrt{Z_c}e^{\int \gamma dx} \qquad (B.14)$$

$$U(x) = \frac{A}{\sqrt{Z_c}} e^{-\int \gamma dx} - \frac{B}{\sqrt{Z_c}} e^{\int \gamma dx} \qquad (B.15)$$

where Z_c is the characteristic impedance of the point of evaluation. The area of validity of this solution is that where the variation of properties along the line is not so great as to cause reflections.

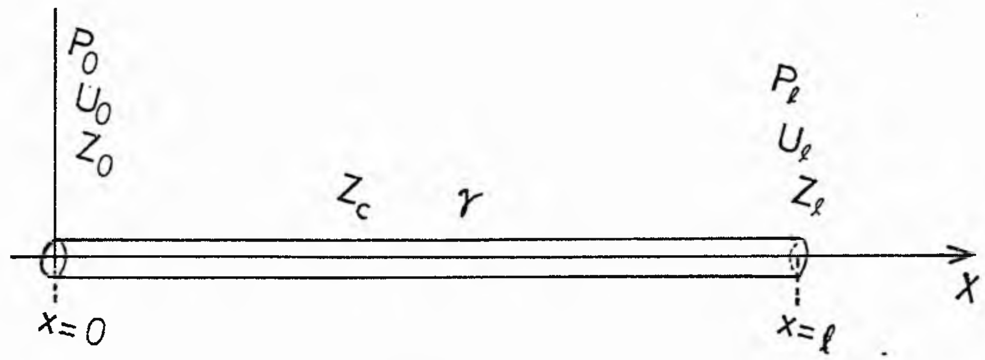

Figure B.1 Acoustic Transmission Line Representation.

APPENDIX C

Manufacturing Processes for Brass Instruments

During fabrication, the various components of brass instruments are subject to various forms of treatment and work. It is the purpose of this section to indicate the general nature and history of this work in order to appreciate the problems involved when trying to control material and structural properties of the finished instrument.

The principal components of the waveguide are a mouthpipe, a length of cylindrical and possibly small sections of flared tubing, and the bell (see figure 1.1).

The most commonly used material is cartridge brass (70% copper, 30% zinc), though a variety of other materials are used - particularly for the bell of the instrument. Table C1 lists some of these materials along with some typical values of their mechanical properties.

An initial problem is being supplied with tubing and sheet metal of consistent constituency. Although cartridge brass is specified to be 30% zinc, its zinc content could be in the range 28-32% (Ross 1972). Also trace impurities will vary from batch to batch.

Much of the cylindrical tubing is used where one piece of tubing must slide within another e.g. a trombone slide, or tuning slides on valved instruments. A requirement is that there must be a good acoustical seal between inner and outer tubes, therefore close tolerances on internal and external tube diameters must be observed. To do this, tubing is bought slightly oversize and precision drawn to the desired dimensions, typically to within 0.0002 inches for the diameter and to within 0.0005 inches for the wall thickness (Boosey and Hawkes Ltd. 1958).

Many tubing components have to be bent through large angles. An old technique is to fill the tube with lead, bending to roughly its final shape, melting the lead out, placing in a split die and forcing metal balls through of successively larger sizes to smooth the tube to the shape of the die. This is now largely superseded by the technique of flattening the tube before bending, bending the tube, and expanding the tube under hydraulic pressure within a split die. Several expansions are usually needed due to work hardening of the brass, which is removed by annealing.

The formation of flared tubing, apart from the larger parts of the bell, is a simple extension of the techniques used on cylindrical tube as described above. The flare is roughly drawn on a mandrel, flattened for rough bending, then expanded in a split die to its final shape.

The smaller parts of the bell are made by the above method, the larger parts being made from sheet metal from which suitable shapes are cut and brazed depending on size and shape of the bell required. This rough shell is then hammered to very nearly the correct shape with particular attention paid to making the seams the same thickness as the original sheet. These bells are then finished over a spinning steel mandrel, against which they are smoothed. While on the spinning mandrel, the large end of the bell is curled back and trimmed to give a strengthened rim of the correct diameter, brass wire usually being inserted in this rim.

There are numerous variants of the above processes: there are several different bell seam configurations, and manufacturers have different forms of rim (e.g. Yamaha sometimes use lead wire; there are trombones with no strengthened rim at all).

Some manufacturers use electroforming techniques to manufacture bells. This is a process by which metal is deposited on a mandrel by an electrolytic process, a process amenable to deposition of alloys as well as elemental metals such as copper.

All the component parts are cleaned off and the instruments assembled, leaving processes like valve grinding or slide fitting till as late as possible to minimize deterioration due to subsequent brazing and work.

There are two facets of the finishing process which are relevant to the present discussion. One is the polishing process which removes a significant amount of material from the instrument, and the other is the fact that a lot of instruments get slightly damaged and dented in the factory and are therefore subject to undefined amounts of work in having these blemishes removed.

MATERIAL	YOUNG'S MODULUS Nm^{-2}	DENSITY $Kg\ m^{-3}$	POISSON'S RATIO
COPPER	$(1.1 \rightarrow 1.24) \times 10^{11}$	$(8.9 \rightarrow 8.96) \times 10^{3}$	0.35
BRASS 70% Cu 30% Zn	$(1.0 \rightarrow 1.17) \times 10^{11}$	$(8.4 \rightarrow 8.8) \times 10^{3}$	0.35
Cu-Be ALLOYS	$(1.24 \rightarrow 1.31) \times 10^{11}$	8.25×10^{3}	-
NICKEL SILVER 60% Cu 25% Zn 15% Ni	1.3×10^{11}	8.7×10^{3}	0.33
NICKEL	2.07×10^{11}	8.9×10^{3}	0.36
BRONZE 95% Cu 5% Sn	1.1×10^{11}	8.8×10^{3}	-
SILVER	7.0×10^{10}	10.5×10^{3}	0.37
STERLING SILVER	7.58×10^{10}	10.5×10^{3}	-

SOURCES : ROSS (1972)
VAN VLACK (1970)
TENNENT (1971)

TABLE C1 PROPERTIES OF SOME MATERIALS

APPENDIX D

COMPUTER PROGRAM LISTINGS

The programs listed in this section are examples of those mentioned in the text plus important subroutines used in these programs.

SIMPLE ON 24/12/80 AT 10:20

```
      DIMENSION NAME(20),ZI(1024),PI(1024)
      COMPLEX T,Z,ZC,GAM
      ACCEPT "IMPEDANCE(0),TRANS. FUNC.(1),CHAR. IMP.(2),OR
     *PROP. COEFF.(3)? ",IMP
      ACCEPT "TUBE LENGTH = ",TLEN
      ACCEPT "TUBE RADIUS = ",TRAD
      ACCEPT "1HZ OR 2HZ STEPS? ",ISTEP
      ACCEPT "MACH NO. OF MEAN FLOW = ",FM
      ACCEPT "TERMINATION TYPE ? OPEN(0), CLOSED(1),
     *NEDERVEEN(2) ",ITYPE
      ACCEPT "FILESIZE ?(6 OR 8 BLOCKS) ",IBLOCK
      TYPE "FILENAME ? "
      READ(11,1)NAME(1)
1     FORMAT(S20)
      CALL FOPEN(1,NAME)
      J=128*IBLOCK
      PH=3.1415926535
      DO 2 I=1,J
      FREQ=I*ISTEP
      TYPE FREQ
      CALL PIZT(TLEN,TRAD,FREQ,ITYPE,T,Z,ZC,GAM,FM)
      IF(IMP.EQ.0)GOTO 3
      IF(IMP.EQ.1)GOTO 4
      IF(IMP.EQ.2)GOTO 5
      ZI(I)=REAL(GAM)
      PI(I)=AIMAG(GAM)
      GOTO 6
5     ZI(I)=CABS(ZC)
      PI(I)=ATAN(AIMAG(ZC)/REAL(ZC))
      GOTO 6
4     ZI(I)=ALOG10(CABS(T))*20.0
      PI(I)=ATAN(AIMAG(T)/REAL(T))
      B=AIMAG(T)
      A=REAL(T)
      IF(A.LT.0.AND.B.GT.0)PI(I)=PI(I)+PH
      IF(A.LT.0.AND.B.LT.0)PI(I)=PI(I)-PH
      GOTO 6
3     ZI(I)=CABS(Z)
      PI(I)=ATAN(AIMAG(Z)/REAL(Z))
6     CONTINUE
2     CONTINUE
      CALL WRBLK(1,0,ZI,IBLOCK,IER)
      CALL WRBLK(1,IBLOCK,PI,IBLOCK,IER)
      CALL FCLOS(1)
      STOP
      END
```

BESSD ON 24/12/80 AT 10:25

```
      SUBROUTINE BESSD(N,M,AR)
      COMPLEX AR,AIR
      AR=1.0/AR
      AIR=CMPLX(0.0,0.0)
      DO 1 I=1,M
      J=M-I
      AIR=1.0/((2.0*(N+J)*AR)-AIR)
1     CONTINUE
      AR=AIR
      RETURN
      END
```

PIZT ON 24/12/80 AT 10:23

```
      SUBROUTINE PIZT(TLEN,TRAD,FREQ,ITYPE,T,Z,ZC,GAM,FM)
      COMPLEX T,Z,GAM,ZC,ZINT,REFL
      CALL PCHAR(FREQ,TRAD,GAM,ZC)
      AM=1.0-(FM**2.0)
      BETA=AIMAG(GAM)
      AZ=(((BETA*TRAD)**2.0)*CABS(ZC))/4.0
      BZ=BETA*TRAD*CABS(ZC)*0.61
      ZINT=CMPLX(AZ,BZ)
      REFL=(ZINT-ZC)/(ZINT+ZC)
      IF(ITYPE.EQ.0)REFL=CMPLX(-1.0,0.0)
      IF(ITYPE.EQ.1)REFL=CMPLX(1.0,0.0)
      ZINT=1.0+(REFL*CEXP(-(2.0*GAM*TLEN/AM)))
      T=(CEXP(-(GAM*TLEN/(1.0+FM)))+REFL*
     *CEXP(-(GAM*TLEN/(1.0+FM))))/ZINT
      Z=ZC*ZINT/(1.0-(REFL*CEXP(-(2.0*GAM*TLEN/AM))))
      RETURN
      END
```

PCHAR ON 24/12/80 AT 10:24

```
      SUBROUTINE PCHAR(FREQ,TRAD,GAM,ZC)
      COMPLEX ZC,GAM,BV2,BV0,BT2,BT0,C0,V,T
      C0=CMPLX(0.0,1.0)
      C1=0.01832
      C2=1.21
      C3=4.2E5
      C4=3.256E5
      C5=1.402
      C6=-1.0
      BV0=CSQRT(C6*C0*C3)*SQRT(FREQ)*TRAD
      BV2=BV0
      BT0=CSQRT(C6*C0*C4)*SQRT(FREQ)*TRAD
      BT2=BT0
      CALL BESSD(1,60,BV0)
      CALL BESSD(1,60,BT0)
      V=1.0-((2.0/BV2)*BV0)
      T=1.0+((C5-1.0)*(2.0/BT2)*BT0)
      ZC=((2.0*C2)/(C1*TRAD*TRAD))*CSQRT(1.0/(T*V))
      GAM=C0*C1*FREQ*CSQRT(T/V)
      RETURN
      END
```

```
      DOUBLE PRECISION F
      DIMENSION Z(256),ZPH(256),T(256),TPH(256),ZU(256),ZUD(256),
    C          TPI(256),TPE(256),ZCALM(256),ZCALPH(256),TCALM(256
    C          TCALPH(256),IDATA(500),V1(250),V2(250),ZNAME(10),
    C          ZCNAME(10),TNAME(10),TCNAME(10),IAT(1),ISW(1),ISF(
      TYPE "PROGRAM TO MEASURE IMPEDANCE AND TRANSFER FUNCTION"
      TYPE "FROM 10HZ TO 1024HZ IN 1HZ STEPS"
      ACCEPT "NO. OF CYCLES = ",NC
      ACCEPT "VELOCITY REFERENCE (CC/S) = ",UR
      ACCEPT "UPPER VELOCITY LIMIT (CC/S) = ",URU
      ACCEPT "LOWER VELOCITY LIMIT (CC/S) = ",URL
      ACCEPT "EXTERNAL MIC. CALIBRATION (PA/V) = ",CALE
      ACCEPT "INTERNAL MIC. CALIBRATION (PA/V) = ",CALI
      TYPE "ANEMOMETER CONSTANTS : "
      ACCEPT "A = ",A," B = ",B," P = ",P,
      ACCEPT "START ATTENUATION (DB) = ",DB
      ACCEPT "DIAMETER OF THROAT (MM) = ",D
      TYPE "FILENAME OF IMPEDANCE CALIBRATION CURVE ?"
      READ(11,1) ZCNAME(1)
 1    FORMAT(S20)
      TYPE "FILENAME OF T.F. CALIBRATION CURVE ?"
      READ(11,1) TCNAME(1)
      TYPE "FILENAME OF Z DATA ?"
      READ(11,1) ZNAME(1)
      TYPE "FILENAME OF T.F. DATA ?"
      READ(11,1) TNAME(1)
      CALL FOPEN(1,ZCNAME)
      CALL FOPEN(2,TCNAME)
      CALL FOPEN(3,ZNAME)
      CALL FOPEN(4,TNAME)
      PI=3.1415926
      IAT(1)=DB*10.0
      ISW(1)=10.0*1638.4
      ISF(1)=0.0
      CALL GIV(7,7,IAT,1,0)
      CALL GIV(0,0,ISW,1,0)
      CALL FREAK(48000.0)
      PAUSE
      N= -2
      M=6
      NFREQ= -256
 2    N=N+2
      M=M+2
      NFREQ=NFREQ+256
      CALL RDBLK(1,N,ZCALM,2,IER)
      CALL RDBLK(1,M,ZCALPH,2,IER)
      CALL RDBLK(2,N,TCALM,2,IER)
      CALL RDBLK(2,M,TCALPH,2,IER)
      DO 3 L=1,256
      IFREQ=NFREQ+L
      IF(IFREQ.LT.10)GO TO 3
      IS=NC*48
 4    CALL GET(2,IDATA(1),IS)
      IP=NC*24
      UT=0.0
      DO 5 I=1,IP
      K=I*2
```

```
      J=K-1
      V1(I)=IDATA(J)/1638.4
      VINT=IDATA(K)/1638.4
      V2(I)=(A+B*VINT*VINT)**(1.0/P)
      UT=UT+V2(I)
5     CONTINUE
      CALL FUNDN(24,NC,V2,URMS,UPH)
      UMS=URMS*PI*D*D/4.0E6
      UCCS=UMS*1.0E6
      AER=UCCS/UR
      DBADD=10.0*ALOG10(AER)
      IF(DBADD.GT.5.0)DBADD=4.5
      IF(DBADD.LT.-5.0)DBADD=-4.5
      DB=DB+DBADD
      IF(DB.LT.0.0)DB=0.0
      IF(DB.GT.59.9)DB=59.9
      IAT(1)=DB*10.0
      IF(UCCS.LT.URL.OR.UCCS.GT.URU) GO TO 6
      CALL GIV(0,0,ISF,1,0)
      UDC=UT/IP
      CALL FUNDN(24,NC,V1,VPRMS,VPPH)
      P1=VPRMS*CALI
      ZUD(L)=UDC
      ZU(L)=UMS
      Z(L)=(P1/UMS)*ZCALM(L)
      ZPH(L)=UPH-VPPH+ZCALPH(L)+PI
30    IF(ZPH(L).GT.PI) ZPH(L)=ZPH(L)-2.0*PI
      IF(ZPH(L).LT.-PI) ZPH(L)=ZPH(L)+2.0*PI
      IF(ZPH(L).GT.PI.OR.ZPH(L).LT.-PI) GO TO 30
      ZMEG=Z(L)/1.0E6
      WRITE(10,40) IFREQ,ZMEG,ZPH(L),P1,UCCS
40    FORMAT(" ",I4," HZ  Z= ",F5.1," MEGS
     CPH= ",F5.2," RAD  PRESS.= ",
     C F5.2," PA  VEL.= ",F6.3," CCS " )
      WRITE(10,50)UDC
50    FORMAT(" MEAN LINEAR VELOCITY = ",F6.3," M/S "/)
      CALL GET(2,IDATA(1),IS)
      CALL GIV(0,0,ISW,1,0)
      F=DBLE((IFREQ+1)*4800.0)
      CALL FREAK(F)
      CALL GIV(7,7,IAT,1,0)
      IP=NC*24
      DO 60 I=1,IP
      K=I*2
      J=K-1
      V1(I)=IDATA(J)/1638.4
      V2(I)=IDATA(K)/1638.4
60    CONTINUE
      CALL FUNDN(24,NC,V1,VIRMS,VIPH)
      CALL FUNDN(24,NC,V2,VERMS,VEPH)
       PRI=VIRMS*CALI
      PE=VERMS*CALE
      T1=PE*TCALM(L)/PRI
      PH=VIPH-VEPH+TCALPH(L)
80    IF(PH.GT.PI)PH=PH-2.0*PI
      IF(PH.LT.-PI)PH=PH+2.0*PI
      IF(PH.GT.PI.OR.PH.LT.-PI)GO TO 80
      TDB=20.0*ALOG10(T1)
      T(L)=TDB
      TPH(L)=PH
```

```
      TPI(L)=PRI
      TPE(L)=PE
      WRITE(10,70)IFREQ,TDB,PH,PRI,PE
 70   FORMAT(" ",I4," HZ  T = ",F5.1," DB
     CPHI = ",F5.2," RAD  P(IN) = ",
     C      F5.2," PA  P(OUT) = ",F5.2," PA"//)
 3    CONTINUE
      CALL WRBLK(3,N,Z,2,IER)
      CALL WRBLK(3,M,ZPH,2,IER)
      CALL WRBLK(3,N+16,ZUD,2,IER)
      CALL WRBLK(3,M+16,ZU,2,IER)
      CALL WRBLK(4,N,T,2,IER)
      CALL WRBLK(4,M,TPH,2,IER)
      CALL WRBLK(4,N+16,PE,2,IER)
      CALL WRBLK(4,M+16,PRI,2,IER)
      IF(N.LT.6) GO TO 2
      CALL FCLOS(1)
      CALL FCLOS(2)
      CALL FCLOS(3)
      CALL FCLOS(4)
      CALL GIV(7,7,500,1,0)
      CALL FREAK(40000,0)
      CALL GIV(0,0,0,1,0)
      GO TO 100
 6    CALL GIV(7,7,IAT,1,0)
      IDELY=1000
      CALL FDELY(IDELY)
      GO TO 4
 100  STOP
      END
```

```
PIT        ON   24/12/80      AT   10:34

      DIMENSION Z(1024),P(1024), NAME(20)
      TYPE "PROGRAM TO DETERMINE C.I. "
      TYPE "FILENAME ? "
      READ(11,1) NAME(1)
 1    FORMAT(S20)
      CALL FOPEN(1,NAME)
      CALL RDBLK(1,0,Z,8,IER)
      CALL RDBLK(1,8,P,8,IER)
      CALL FCLOS(1)
      ACCEPT "START IMP. = ",ZT
      DO 2 I=10,1024
      Z(I)=ALOG(Z(I))
 2    CONTINUE
      ZT=ALOG(ZT)
 3    CONTINUE
      ZS=0.0
      DO 4 I=10,1024
      ZS=ZS+Z(I)/ZT
 4    CONTINUE
      ZS=ZS/1015.0
      TEST=1.0-ZS
      ZP=EXP(ZT)
      TYPE "C.I. = ",ZP,"   INC. = ",TEST
      ZP=ZP-(ZP*TEST*15.0)
      ZT=ALOG(ZP)
      IF(ABS(TEST).GT.0.000001)GO TO 3
      STOP
      END
```

FEDUP ON 19/01/80 AT 11:34

```
      DIMENSION VCOD(340,3),NAME1(20),TH(21),DA(16),VMODE(340,6),EAR(21),
     CVDIS(384),P(21),AMASS(36),ASTIFF(36),AFRM(36),MANE(20),OP1(1024)
     C,OP2(1024),NAMA(20),PAMP(1024),OP(16)
      DIMENSION OP3(128),OP4(128),OP5(128),OP6(128),OP7(128),OP8(128)
     *,OP15(128),OP16(128),VDISN(384)
      COMMON/C1/ VCOD,NAME1,NN,NM,TH,DA,D,PR,E,PI,JR,MN,VMODE,FRM,EAR,
     CVDIS,GM,GS,FRC,P,GRM,GRP,AJ,GC,PA,FREQ,GD
      COMMON/C2/ RAREA,VDISN
      TYPE "      ******************************************************"
      TYPE "      ******************************************************"
      TYPE "      ** FINITE                                          **"
      TYPE "      **  ELEMENT                                        **"
      TYPE "      **   DYNAMIC                                       **"
      TYPE "      **    UNILATERAL                                   **"
      TYPE "      **     PROCESSOR                                   **"
      TYPE "      ******************************************************"
      TYPE "      ******************************************************"
      CALL POP
      CALL DATIN
      CALL AREA
      TYPE EAR
      ACCEPT "NO. OF MODES TO BE ACCESSED = ",NMA
      CALL CFILW("ABCDEF",2,IERR)
      CALL FOPEN(2,"ABCDEF")
      DO 1 MN=1,NMA
      CALL GETMODE
      CALL MASS
      AMASS(MN)=GM
      TYPE "GEN. MASS = ",GM
      CALL FSTIFF
      ASTIFF(MN)=GS
      CALL NORM
      IB=3*MN-3
      CALL WRBLK(2,IB,VDIS,3,IER)
      IB1=IB+50
      CALL WRBLK(2,IB1,VDISN,3,IER)
      AFRM(MN)=FRM
       TYPE "FREQ = ",FRM
1     CONTINUE
      ACCEPT "START FREQUENCY = ",FSA
      ACCEPT "FREQ. STEP = ",FSP
      ACCEPT "HIGHEST FREQUENCY = ",UFR
      TYPE "STORE FILENAME ? "
      READ(11,7)MANE(1)
7     FORMAT(S20)
      CALL FOPEN(4,MANE)
      DO 2 I=1,1024
      OPJ=1.0E-70
      OPI=1.0E-70
      FI=I
      FREQ=FSA+(FI*FSP)
      IF(FREQ.GT.UFR)GO TO 9
      IF(FREQ.LT.51.0)GO TO 9
      CALL GETPROF
      PA=1.0
      DO 3 MN=1,NMA
      IB=3*MN-3
```

```
      CALL RDBLK(2,IB,VDIS,3,IER)
      IB1=IB+50
      CALL RDBLK(2,IB1,VDISN,3,IER)
      FRM=AFRM(MN)
      GM=AMASS(MN)
      GS=ASTIFF(MN)
      CALL DAMP
      CALL RESP
      OPI=OPI+GC
3     CONTINUE
      ARM=2.0*PI*FREQ
      OPI=ABS(OPI*ARM)
      OPJ=ABS(OPJ*ARM)
9     CONTINUE
      OP1(I)=OPI
2     CONTINUE
      CALL WRBLK(4,0,OP1,8,IER)
      CALL FCLOS(2)
      CALL DFILW("ABCDEF",IER)
      CALL FCLOS(4)
      STOP STARING AND SWITCH ME OFF!!!!!!!!!
      END
```

```
MASS       ON   19/01/80      AT   11:35

      SUBROUTINE MASS
      DIMENSION VCOD(340,3),NAME1(20),TH(21),DA(16),VMODE(340,6),EAR(21),
     CVDIS(384),P(21)
      COMMON/C1/ VCOD,NAME1,NN,NM,TH,DA,D,PR,E,PI,JR,MN,VMODE,FRM,EAR,
     CVDIS,GM,GS,FRC,P,GRM,GRP,AJ,GC,PA,FREQ,GD
      COMMON/C2/ RAREA
      TYPE "STAGE 4 -- CALCULATE GENERALISED MASS "
      IC=0
      GM=0.0
      DO 1 I=1,JR
      DO 2 J=1,16
      IC=IC+1
      B=VMODE(IC,1)*VMODE(IC,1)
      C=VMODE(IC,2)*VMODE(IC,2)
      F=VMODE(IC,3)*VMODE(IC,3)
      A=B+C+F
      K=I-1
      IF(I.EQ.1)K=I
      L=I+1
      IF(I.EQ.JR)L=I
      GM=GM+(D*((TH(I)+TH(K)+TH(L))/3.0)*EAR(I)*A)
      IF(I.NE.JR)GO TO 2
      GM=GM+(D*A*RAREA*PI*VCOD(IC,1)/8.0)
2     CONTINUE
1     CONTINUE
      TYPE "STAGE 4 COMPLETED "
      RETURN
      END
```

AREA ON 19/01/80 AT 11:35

```
      SUBROUTINE AREA
      DIMENSION VCOD(340,3),NAME1(20),TH(21),DA(16),VMODE(340,6),EAR(21),
     CVDIS(384),P(21)
      COMMON/C1/ VCOD,NAME1,NN,NM,TH,DA,D,PR,E,PI,JR,MN,VMODE,FRM,EAR,
     CVDIS,GM,GS,FRC,P,GRM,GRP,AJ,GC,PA,FREQ,GD
      TYPE "STAGE 2 - CALCULATE EFFECTIVE AREAS "
      DO 6 I=1,JR
      EAR(I)=0.0
 6    CONTINUE
      IC=-15
      DO 1 K=1,JR
      IC=IC+16
      IF(K.EQ.1)GOTO 3
C   CYLINDER OR CONE APPROX. Y- SIDE ?
      A=(VCOD(IC,1)-((VCOD(IC,1)+VCOD((IC-16),1))/2.0))/(((VCOD(IC,2)-
     CVCOD((IC-16),2))/2.0))
      IF(A.GT.0.01)GO TO 2
C   CYLINDER APPROX.
      EAR(K)=2.0*PI*VCOD(IC,1)*(VCOD(IC,2)-VCOD((IC-16),2))/2.0
      GO TO 3
 2    CONTINUE
C   CONE APPROX.
      A=(VCOD(IC,1)**2.0-((VCOD(IC,1)+VCOD((IC-16),1))/2.0)**2.0)
      B=SIN(ATAN((VCOD(IC,1)-((VCOD(IC,1)+VCOD((IC-16),1))/2.0))/(((VCOD(I
     CVCOD((IC-16),2))/2.0))))
      EAR(K)=PI*A/B
 3    CONTINUE
      IF(K.EQ.JR)GO TO 5
C   CYLINDER OR CONE APPROX. Y+ SIDE ?
      A=(((VCOD((IC+16),1)+VCOD(IC,1))/2.0)-VCOD(IC,1))/(((VCOD((IC+16
     C),2)-VCOD(IC,2))/2.0)-
     C0.0)
      IF(A.GT.0.01)GO TO 4
C   CYLINDER APPROX.
      EAR(K)=EAR(K)+(2.0*PI*VCOD(IC,1)*((VCOD((IC+16),2)-VCOD(IC,2))/2.0)
      GO TO 5
 4    CONTINUE
C   CONE APPROX.
      A=(((VCOD((IC+16),1)+VCOD(IC,1))/2.0)**2.0-VCOD(IC,1)**2.0)
      B=SIN(ATAN((((VCOD((IC+16),1)+VCOD(IC,1))/2.0)-VCOD(IC,1))/(((VC
     COD((IC+16),2)-
     CVCOD(IC,2))/2.0))))
      EAR(K)=EAR(K)+(PI*A/B)
 5    CONTINUE
      EAR(K)=EAR(K)/16.0
 1    CONTINUE
      TYPE "STAGE 2 COMPLETED"
      RETURN
      END
```

GETMODE ON 19/01/80 AT 11:35

```
      SUBROUTINE GETMODE
      DIMENSION VCOD(340,3),NAME1(20),TH(21),DA(16),VMODE(340,6),EAR(21),
     CVDIS(384),P(21),DING(2048)
      COMMON/C1/ VCOD,NAME1,NN,NM,TH,DA,D,PR,E,PI,JR,MN,VMODE,FRM,EAR,
     CVDIS,GM,GS,FRC,P,GRM,GRP,AJ,GC,PA,FREQ,GD
      TYPE "STAGE 1A - READ MODE ",MN
      CALL FOPEN(1,NAME1)
      NB=(MN-1)*16
      CALL RDBLK(1,NB,DING,16,IER)
      FRM=DING(2048)
      DO 1 I=1,NN
      VMODE(I,1)=DING(I)
      VMODE(I,2)=DING(I+NN)
      VMODE(I,3)=DING(I+(2*NN))
      VMODE(I,4)=DING(I+(3*NN))
      VMODE(I,5)=DING(I+(4*NN))
      VMODE(I,6)=DING(I+(5*NN))
 1    CONTINUE
      CALL FCLOS(1)
      TYPE "STAGE 1A COMPLETED "
      RETURN
      END
```

GETPROF ON 19/01/80 AT 11:37

```
      SUBROUTINE GETPROF
      DIMENSION VCOD(340,3),NAME1(20),TH(21),DA(16),VMODE(340,6),EAR(21),
     CVDIS(384),P(21),PRI(128)
      COMMON/C1/ VCOD,NAME1,NN,NM,TH,DA,D,PR,E,PI,JR,MN,VMODE,FRM,EAR,
     CVDIS,GM,GS,FRC,P,GRM,GRP,AJ,GC,PA,FREQ,GD
      TYPE "STAGE 1B - READ PROFILE ",FREQ," HZ"
      CALL FOPEN(1,"PROFILE")
      AFR=FREQ-51.0
      IFR=IFIX(AFR/5.0)
      CALL RDBLK(1,IFR,PRI,1,IER)
      CALL FCLOS(1)
      JFR=(AFR-(IFR*5))
      DO 1 I=1,21
      IJ=I+(JFR*21)
      P(I)=PRI(IJ)
 1    CONTINUE
      TYPE "STAGE 1B COMPLETED"
      RETURN
      END
```

NORM ON 19/01/80 AT 11:36

```
      SUBROUTINE NORM
      DIMENSION VCOD(340,3),NAME1(20),TH(21),DA(16),VMODE(340,6),EAR(21),
     CVDIS(384),P(21),VDISN(384)
      COMMON/C1/ VCOD,NAME1,NN,NM,TH,DA,D,PR,E,PI,JR,MN,VMODE,FRM,EAR,
     CVDIS,GM,GS,FRC,P,GRM,GRP,AJ,GC,PA,FREQ,GD
      COMMON/C2/ RAREA,VDISN
      TYPE "STAGE 3 -- CALCULATE DISPLACEMENTS NORMAL TO BELL SURFACE "
      IC=0
      DO 1 I=1,JR
      DO 2 J=1,16
      IC=IC+1
      X=VMODE(IC,1)
      Z=VMODE(IC,3)
      RAD=SQRT((X*X)+(Z*Z))
      THETA=ATAN(X/Z)
      IF(Z.LT.0.AND.X.GT.0)THETA=THETA+PI
      IF(Z.LT.0.AND.X.LE.0)THETA=THETA-PI
      VDISN(IC)=RAD*COS(VCOD(IC,3)-THETA)
      IF(I.EQ.1)GO TO 3
      IF(I.EQ.JR)GO TO 4
      X=VCOD((IC+16),1)-VCOD((IC-16),1)
      Y=VCOD((IC+16),2)-VCOD((IC-16),2)
 5    CONTINUE
      PHI=ATAN(X/Y)
      IF(Y.LT.0.AND.X.GT.0)PHI=PHI+PI
      IF(Y.LT.0.AND.X.LT.0)PHI=PHI-PI
      VDIS(IC)=VDISN(IC)*COS(PHI)+VMODE(IC,2)*COS(PHI+PI/2.0)
      GO TO 2
 3    CONTINUE
      VDIS(IC)=0.0
      VDISN(IC)=0.0
      GO TO 2
 4    CONTINUE
      X=4.0*VCOD(IC,1)-VCOD((IC-16),1)
      Y=2.0*(VCOD(IC,2)-VCOD((IC-16),2))
      GO TO 5
 2    CONTINUE
 1    CONTINUE
      TYPE "STAGE 3 COMPLETED"
      RETURN
      END
```

FSTIFF ON 19/01/80 AT 11:36

```
      SUBROUTINE FSTIFF
      DIMENSION VCOD(340,3),NAME1(20),TH(21),DA(16),VMODE(340,6),EAR(21),
     CVDIS(384),P(21)
      COMMON/C1/ VCOD,NAME1,NN,NM,TH,DA,D,PR,E,PI,JR,MN,VMODE,FRM,EAR,
     CVDIS,GM,GS,FRC,P,GRM,GRP,AJ,GC,PA,FREQ,GD
      TYPE "STAGE 5A - CALCULATE IMPLIED STIFFNESS"
      GS=((2.0*PI*FRM)**2.0)*GM
      TYPE "STAGE 5A COMPLETED"
      RETURN
      END
```

RESP ON 19/01/80 AT 11:38

```
      SUBROUTINE RESP
      DIMENSION VCOD(340,3),NAME1(20),TH(21),DA(16),VMODE(340,6),EAR(21),
     CVDIS(384),F(21)
      COMMON/C1/ VCOD,NAME1,NN,NM,TH,DA,D,PR,E,PI,JR,MN,VMODE,FRM,EAR,
     CVDIS,GM,GS,FRC,F,GRM,GRP,AJ,GC,PA,FREQ,GD
      COMPLEX Q,GR
      AF=2.0*PI*FREQ
      Q=CMPLX((GS-GM*AF*AF),(AF*GD))
      GR=1.0/Q
      GRM=CABS(GR)
      A=REAL(GR)
      B=AIMAG(GR)
      GRP=(B/A)
      IF(A.LT.0.AND.B.GT.0)GRP=GRP+PI
      IF(A.LT.0.AND.B.LE.0)GRP=GRP-PI
      IC=0
      AJ=0.0
      DO 1 I=1,JR
      DO 2 J=1,16
      IC=IC+1
      AJ=AJ+(F(I)*VDIS(IC)*EAR(I))
    2 CONTINUE
    1 CONTINUE
      AJ=ABS(AJ)
      GC=GRM*PA*AJ
      RETURN
      END
```

DAMP ON 19/01/80 AT 11:38

```
      SUBROUTINE DAMP
      DIMENSION VCOD(340,3),NAME1(20),TH(21),DA(16),VMODE(340,6),EAR(21),
     CVDIS(384),F(21)
      COMMON/C1/ VCOD,NAME1,NN,NM,TH,DA,D,PR,E,PI,JR,MN,VMODE,FRM,EAR,
     CVDIS,GM,GS,FRC,F,GRM,GRP,AJ,GC,PA,FREQ,GD
      FL=(-7.52E-07*FREQ)+1.1E-03
      WF=2.0*PI*FREQ
      WM=2.0*PI*FRM
      GD=WM*WM*GM*FL/WF
      RETURN
      END
```